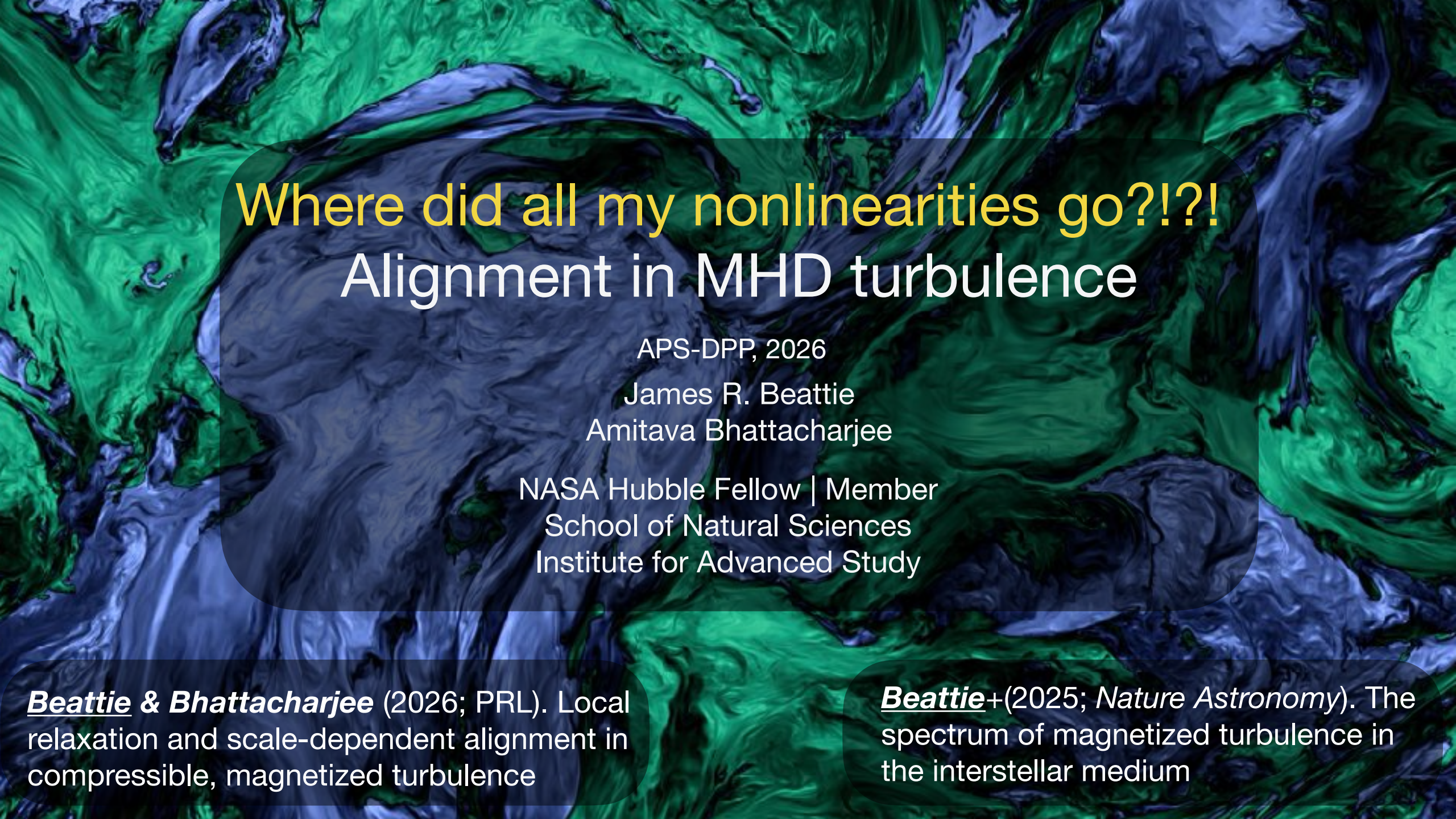


Where did all my nonlinearities go?!?!
Alignment in MHD turbulence

APS-DPP, 2026

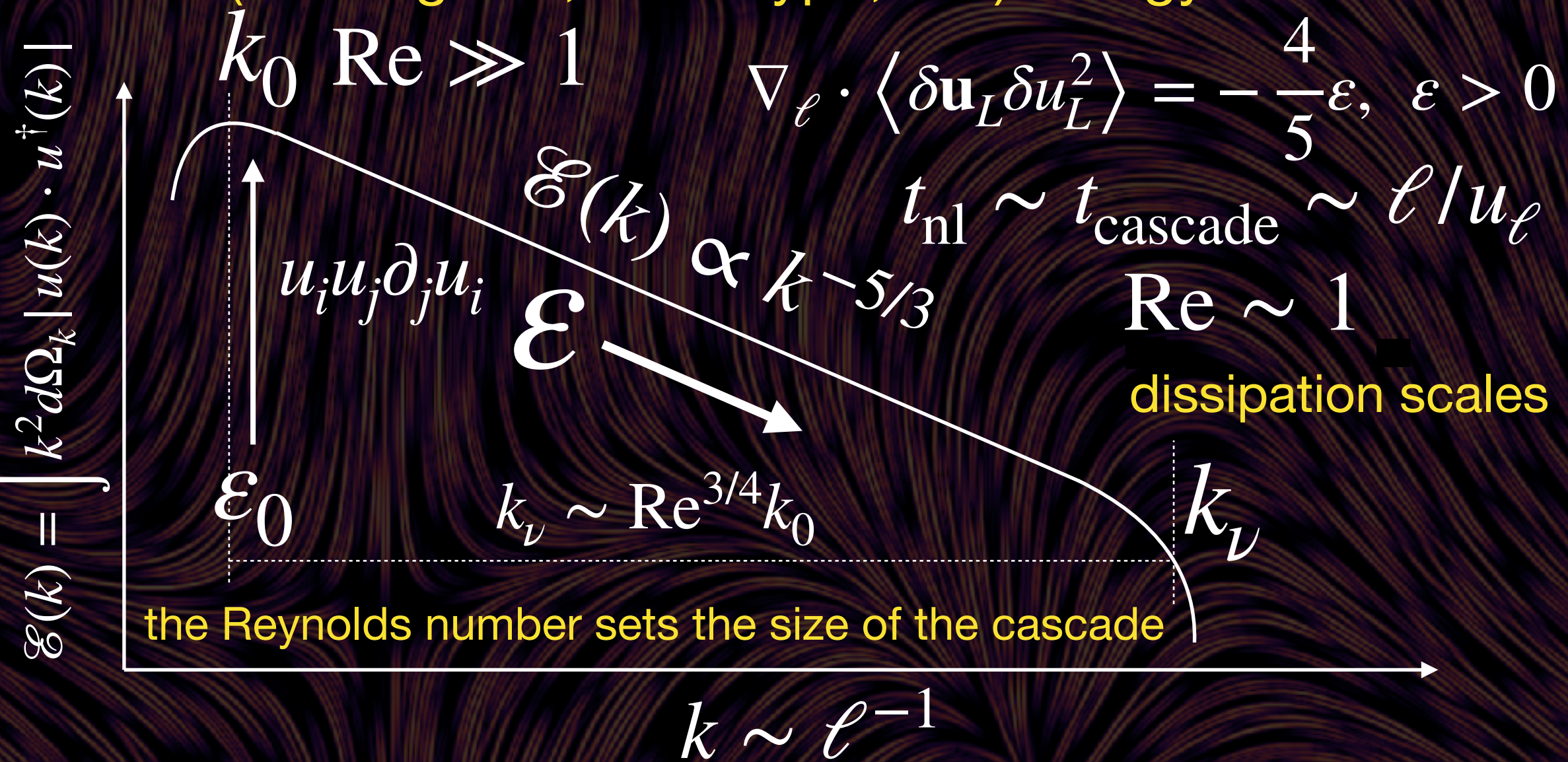
James R. Beattie
Amitava Bhattacharjee

NASA Hubble Fellow | Member
School of Natural Sciences
Institute for Advanced Study

Beattie* & *Bhattacharjee (2026; PRL). Local relaxation and scale-dependent alignment in compressible, magnetized turbulence

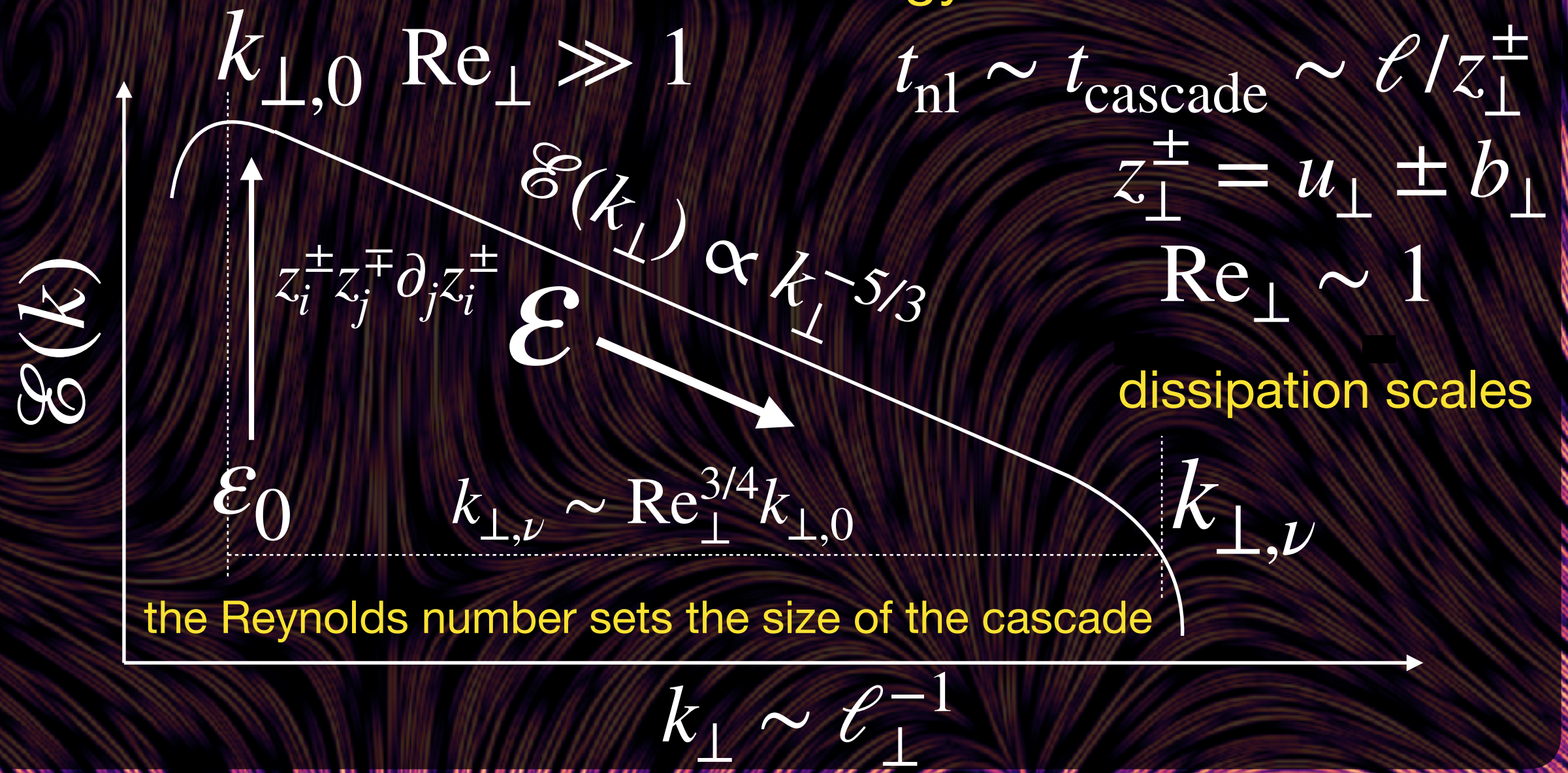
Beattie+(2025; *Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

The (Kolmogorov, 1941-type; HIT) energy cascade



Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The dream MHD energy cascade

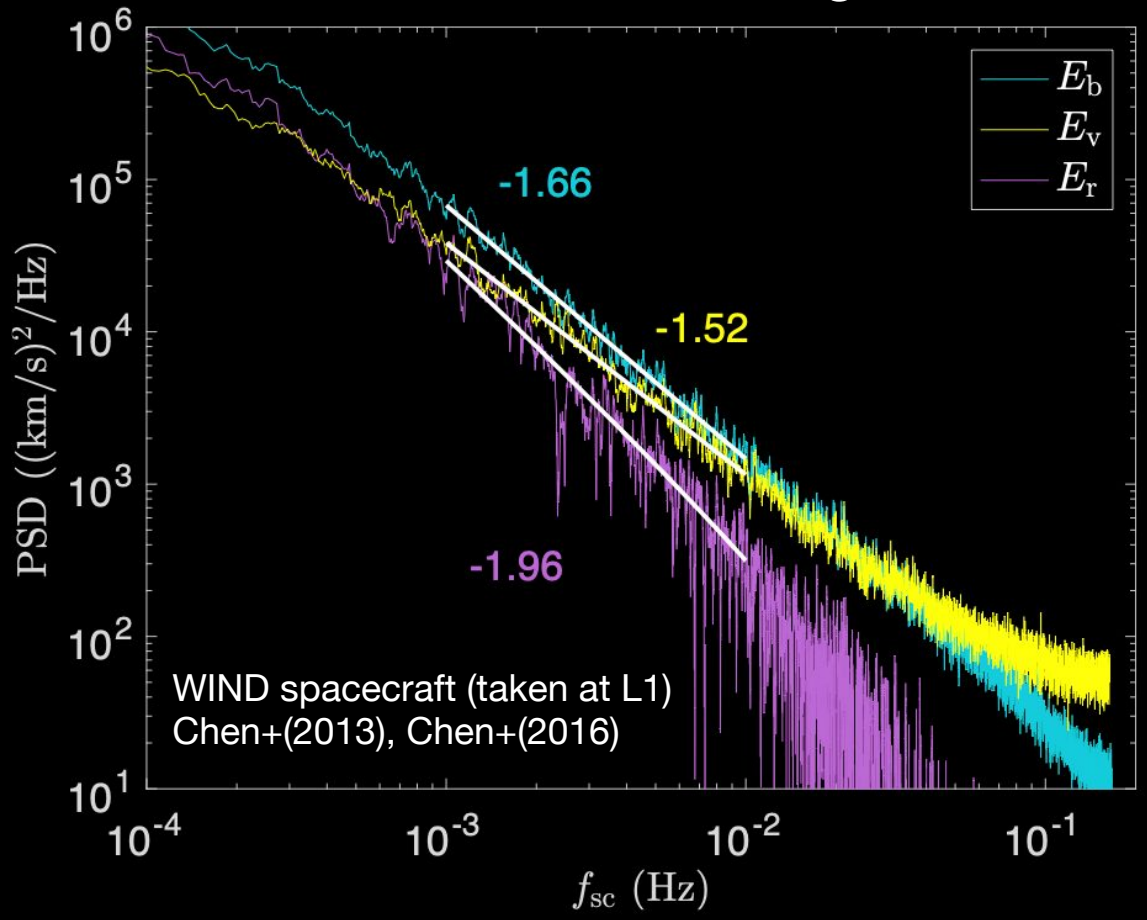


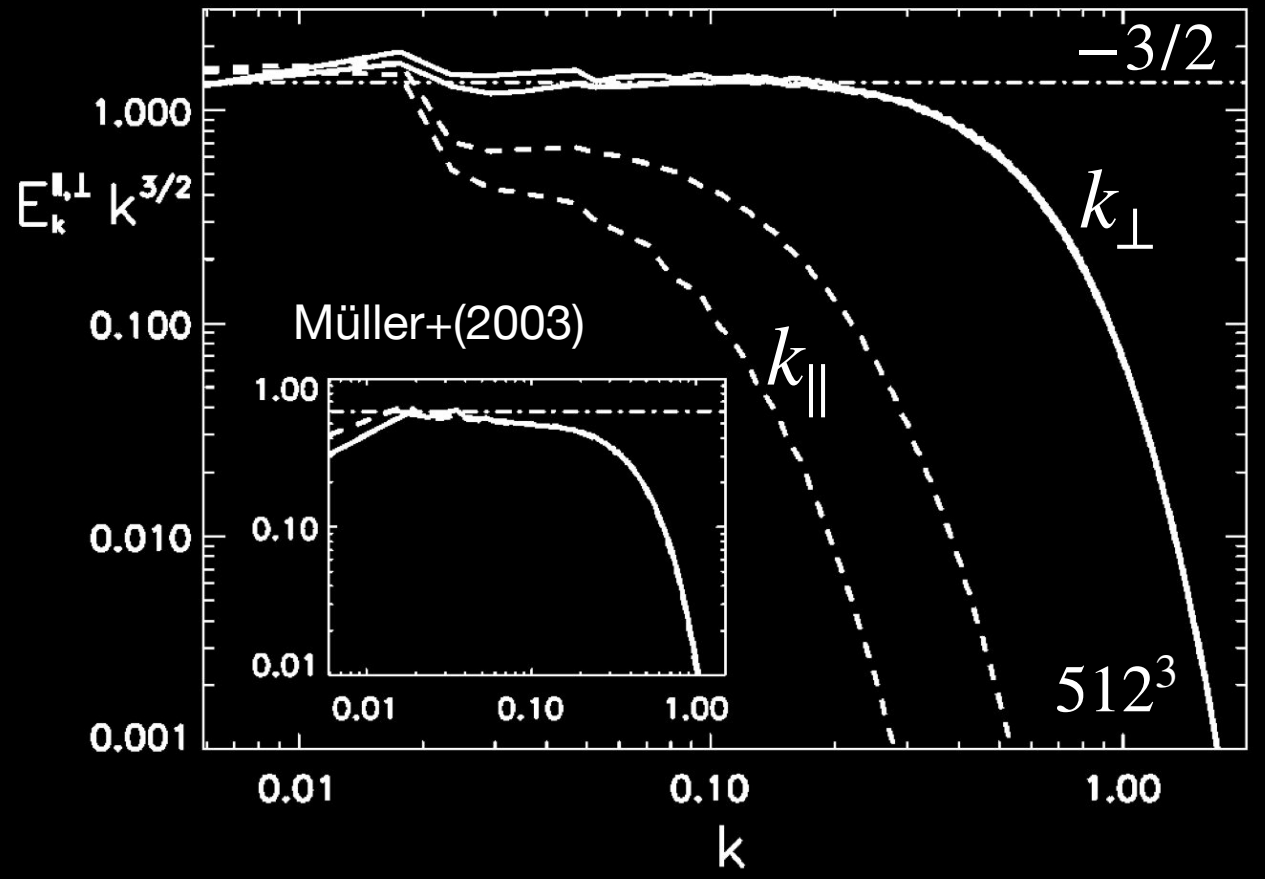
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The incompressible MHD energy cascade

Goldreich & Shridar (1995)??

High-resolution simulations and solar wind observations



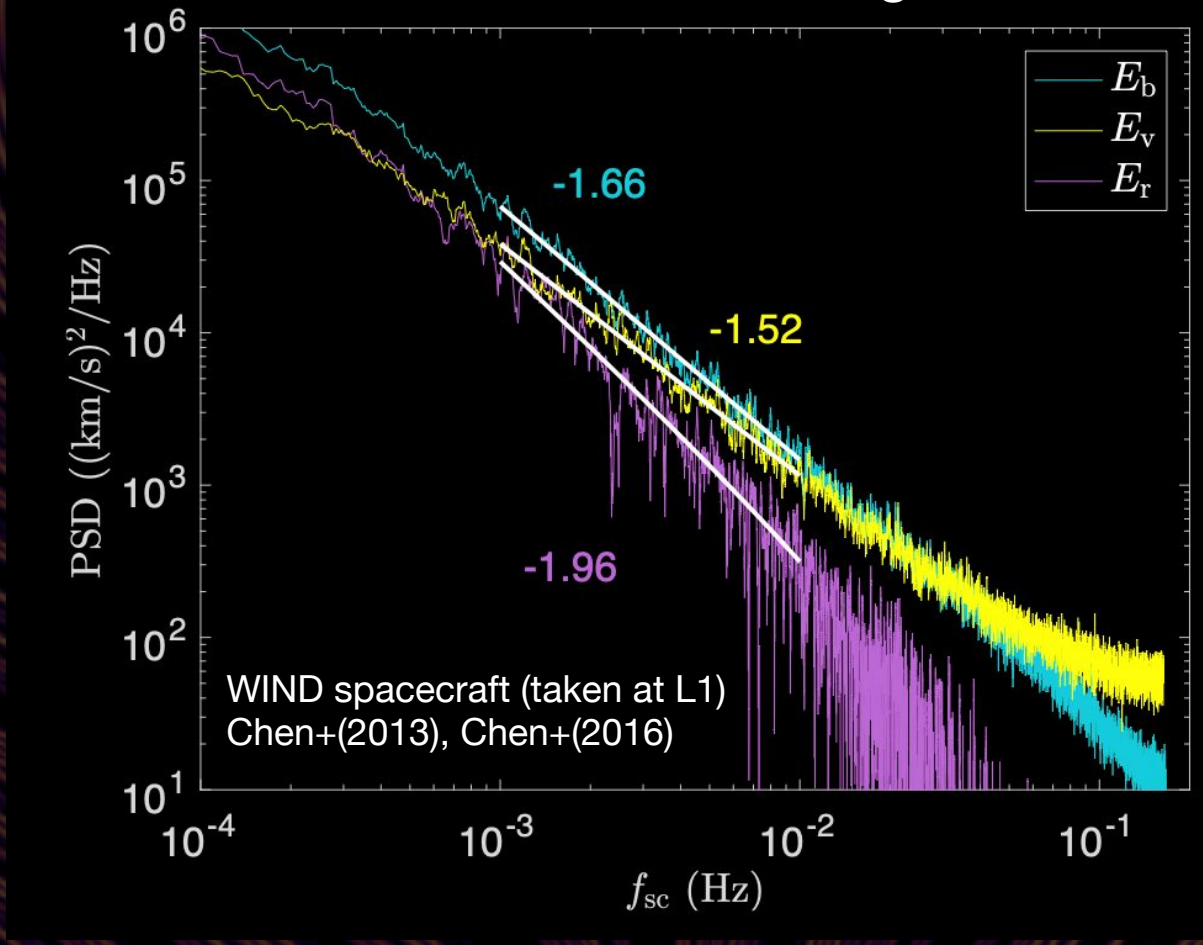


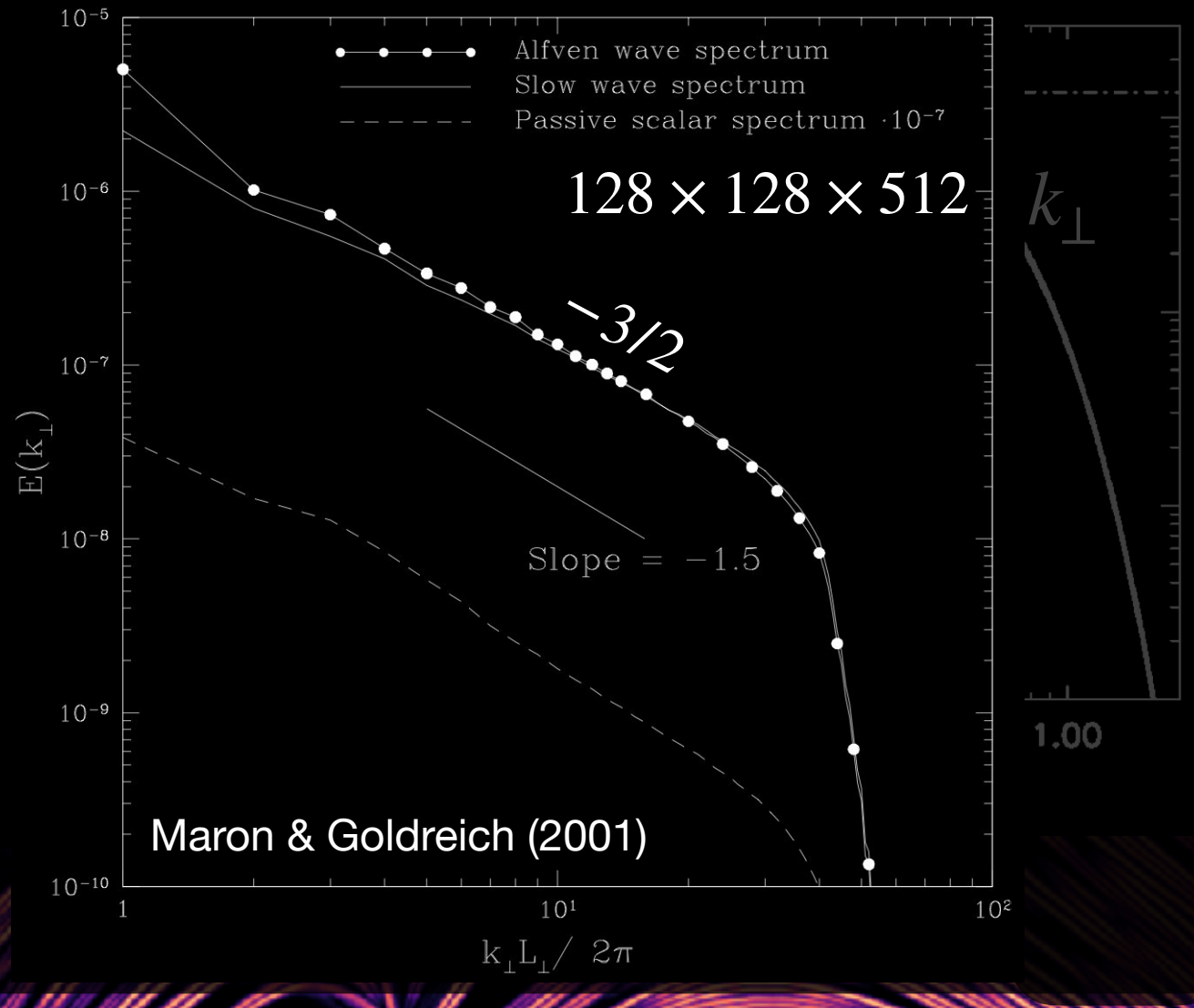
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Goldreich & Shridar (1995)??

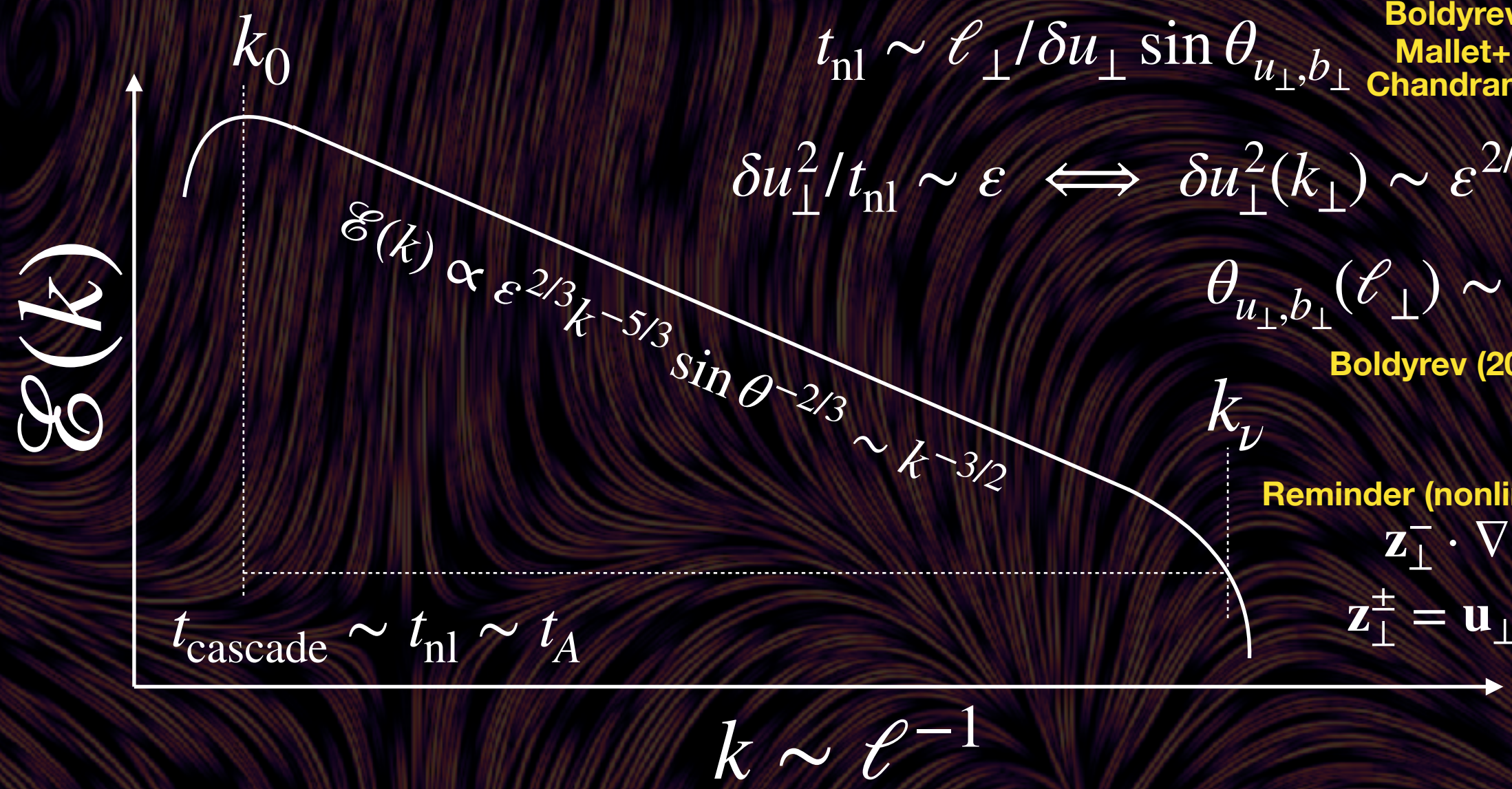
High-resolution simulations and solar wind observations





Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

How to deplete the nonlinearities? Dynamic alignment!



$$t_{\rm nl} \sim \ell_\perp / \delta u_\perp \sin\theta_{u_\perp, b_\perp}$$

Boldyrev (2006)
Mallet+ (2015)
Chandran+ (2015)

$$\delta u_\perp^2 / t_{\rm nl} \sim \varepsilon \iff \delta u_\perp^2(k_\perp) \sim \varepsilon^{2/3} k_\perp^{-3/2}$$

$$\theta_{u_\perp, b_\perp}(\ell_\perp) \sim \ell_\perp^{1/4}$$

Boldyrev (2006)

Reminder (nonlinearity):

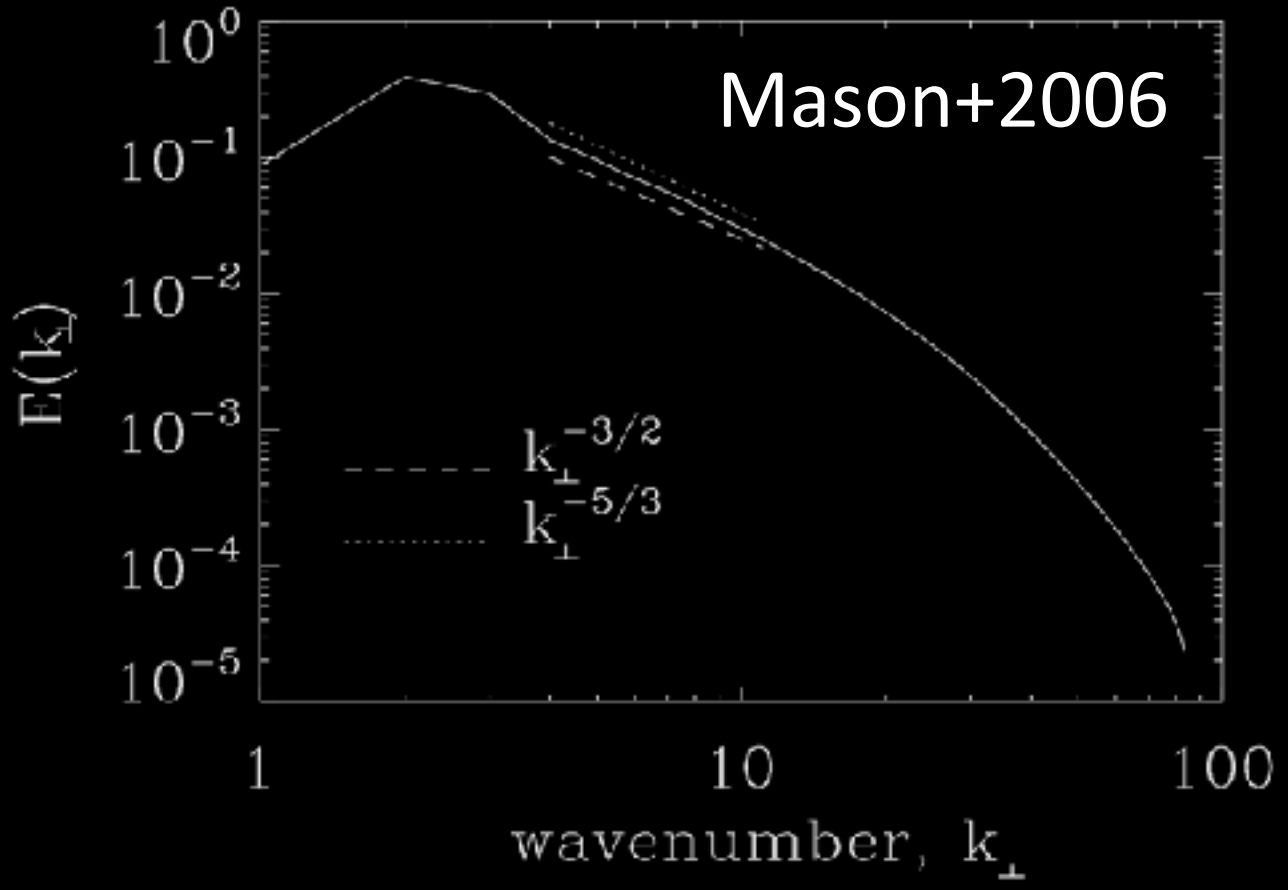
$$\mathbf{z}_\perp^- \cdot \nabla \otimes \mathbf{z}_\perp^+$$

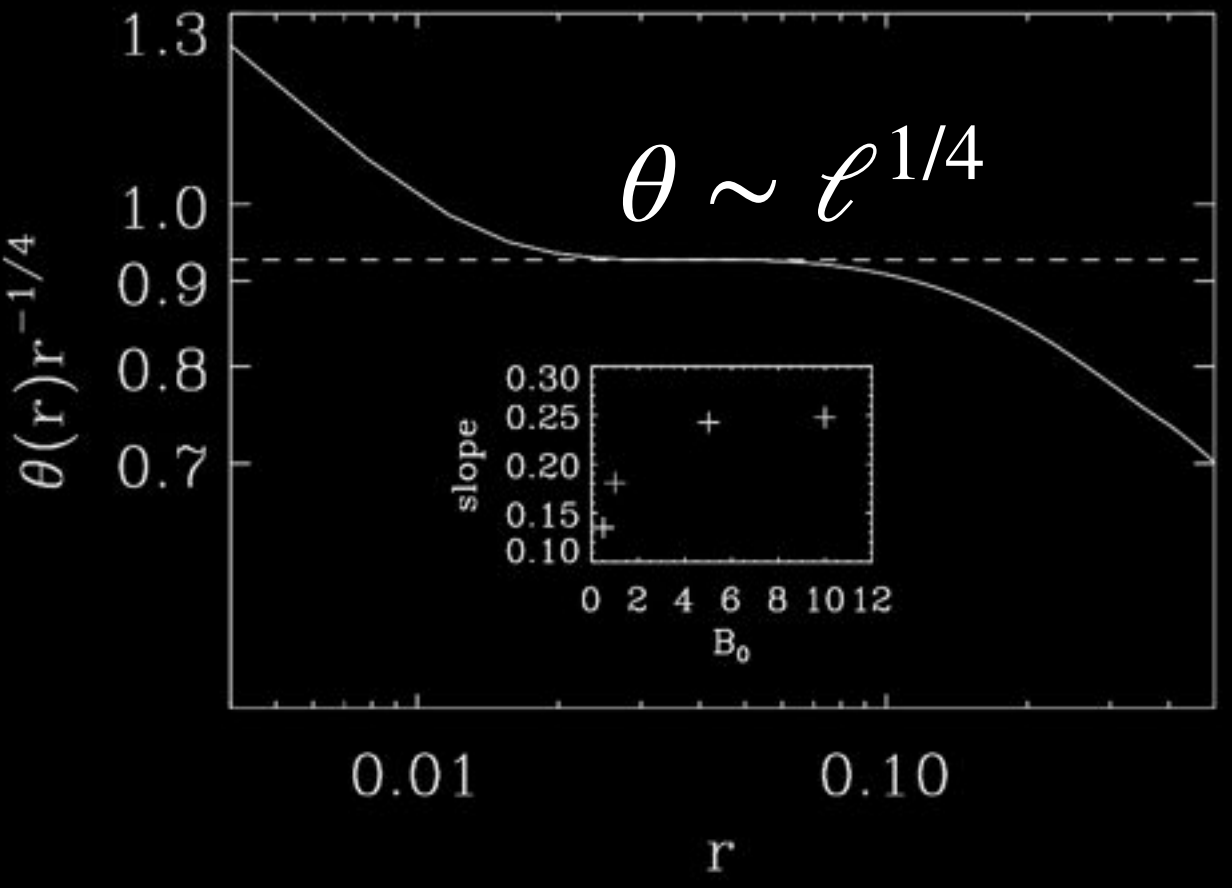
$$\mathbf{z}_\perp^\pm = \mathbf{u}_\perp \pm \mathbf{b}_\perp$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

How to deplete the nonlinearities? Dynamic Alignment!

Boldyrev (2006)





$$k \sim \ell^{-1}$$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

An emergent asymptotic regime?

$\mathcal{E}(k_\perp)$

$\theta_{\boldsymbol{u},\boldsymbol{b}} \sim \ell_\perp/\xi \sim \ell_\perp^{1/4}$

$k_\perp^{-3/2}$ sheet-like anisotropy

Boldyrev (2006)

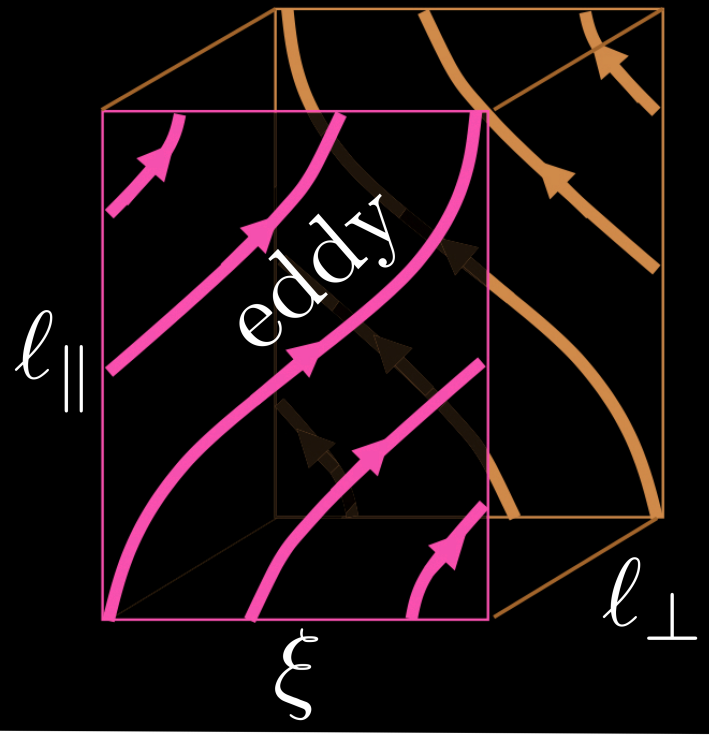
$t_{\rm nl} \sim (\gamma_{\rm tearing})^{-1}$

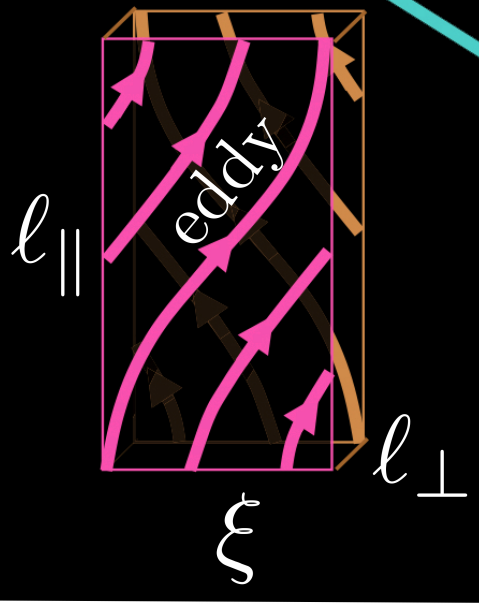
Loureiro & Boldyrev (2017)
Comisso et al. (2018)
Dong et al. (2022)

tearing unstable

k_*

$k_\perp^{-11/5}$







$k_\perp$

$k_* L \sim \mathrm{Rm}_{\rm crit}^{4/7}, \quad \mathrm{Rm}_{\rm crit} \approx 10^5$ Dong+ (2022)

Magnetized compressible turbulence on $10{,}000^3$ grids

Beattie, Federrath, Klessen, Cielo & Bhattacharjee

1. What is the scaling of the energy cascade in highly-compressible MHD turbulence?
2. **How are the characteristic scales organized in the compressible turbulence?**
3. **What are the saturation physics of the turbulent dynamo?**

PI of a 160million core-hour project on superMUC-NG

ILES of compressible MHD turbulence

Turbulence: $\sigma_V/c_s \approx 4,\ \ell_0 = L/2$

Magnetic fields: $B = b_{\rm turb},\ B = B_0,\ \mathcal{M}_{\rm A} \approx 2$

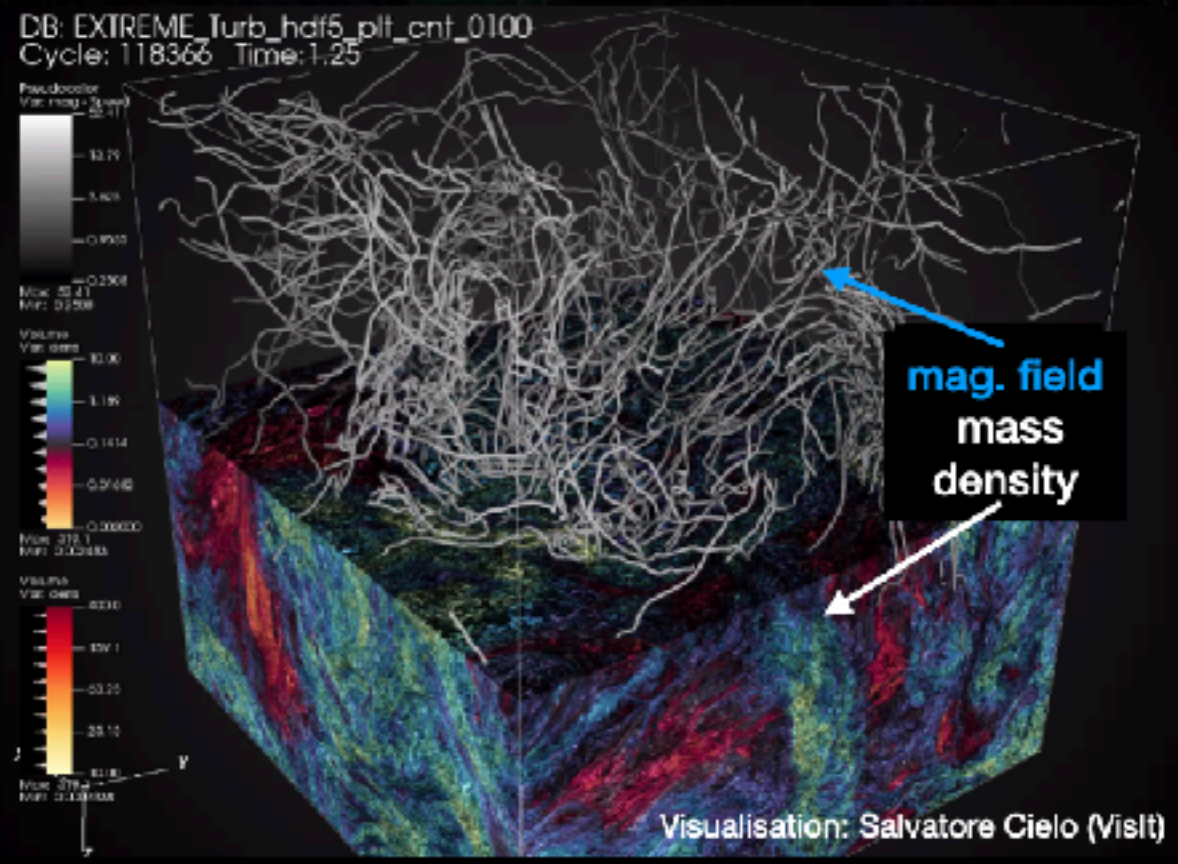
Three experiments for convergence tests:

- **LOW-RES:** $\approx 2{,}500^3$ **(0.3Mcore-h, 8,640cores)**
- **MID-RES:** $\approx 5{,}000^3$ **(4.0Mcore-h, 34,560cores)**
- **HIGH-RES:** $\approx 10{,}000^3$ **(80.0Mcore-h, 148,240cores)**
  - **Factor of 4 higher linear grid resolution than IllustrisTNG**

≈ 6.00PB **in data products**

Broad Code details:

- Highly-modified version of finite volume code *FLASH*, second-order in space approximate Riemann (PPM) solver with framework outlined in Bouchut+2010, tested in *FLASH* in Waagen+2011.
- Ideal (ILES) compressible non-helical, isothermal MHD turbulence with finite correlation time (OU process; Federrath2022).



The velocity power spectrum at high-k

$$\mathscr{E}(k) \sim k^{-3/2},\ k \gtrsim k_{\mathrm{eq}}$$

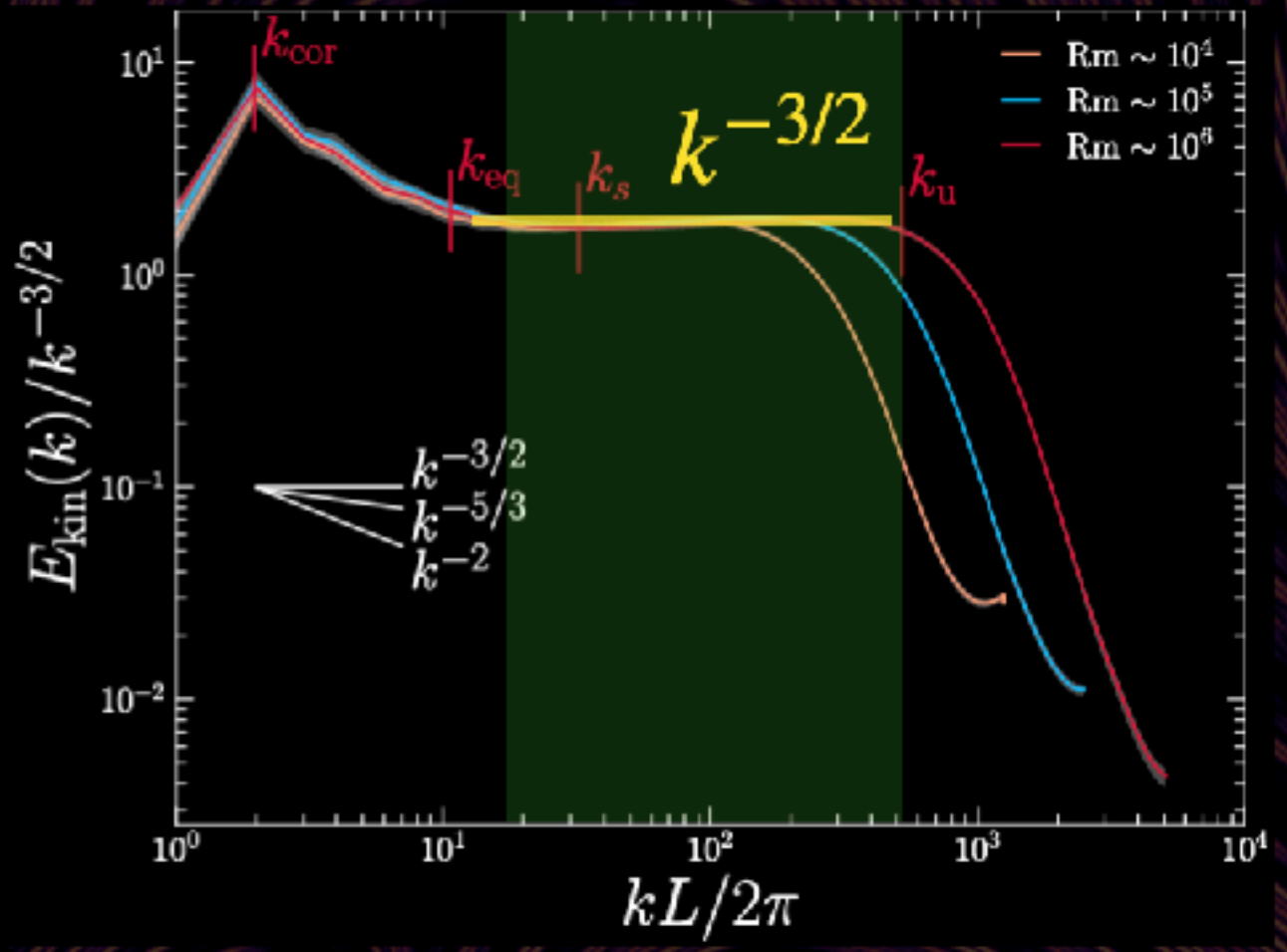
Quasi-incompressible on these scales since steep k^{-2} $|\nabla \times \mathbf{u}| = 0$ mode spectrum.

$$|u_{\mathrm{comp}}| \ll |u_{\mathrm{sol}}|$$

And Alfvénic to sub-Alfvénic

$$\delta u \sim \delta b$$

Perfect conditions for **Alfvén mode theories**. So, dynamical alignment?

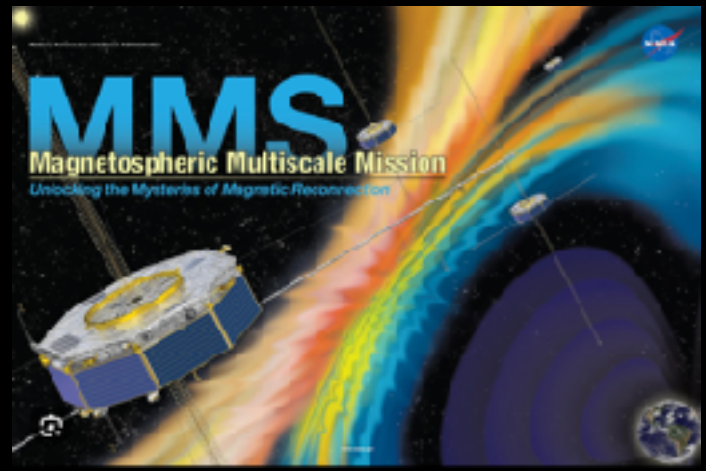


Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Alignment PDFs: simulations + MMS data

Pecora et al. (2023) for MMS data.

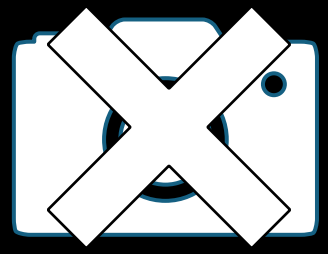
Beattie* & *Bhattacharjee (2026; PRL). Local relaxation and scale-dependent alignment in compressible, magnetized turbulence

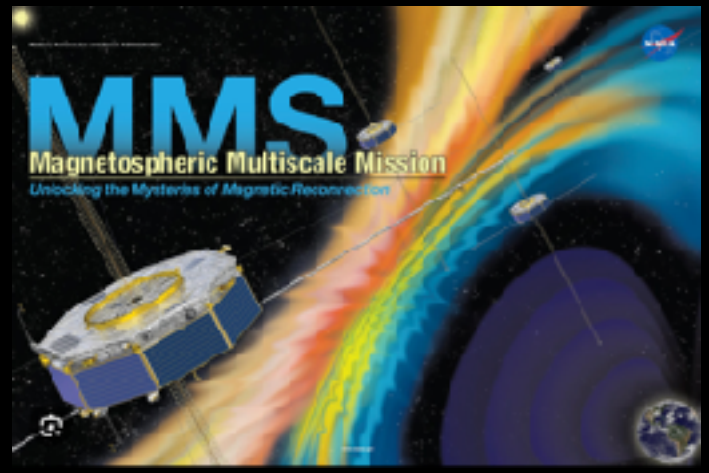


Alignment PDFs: simulations + MMS data

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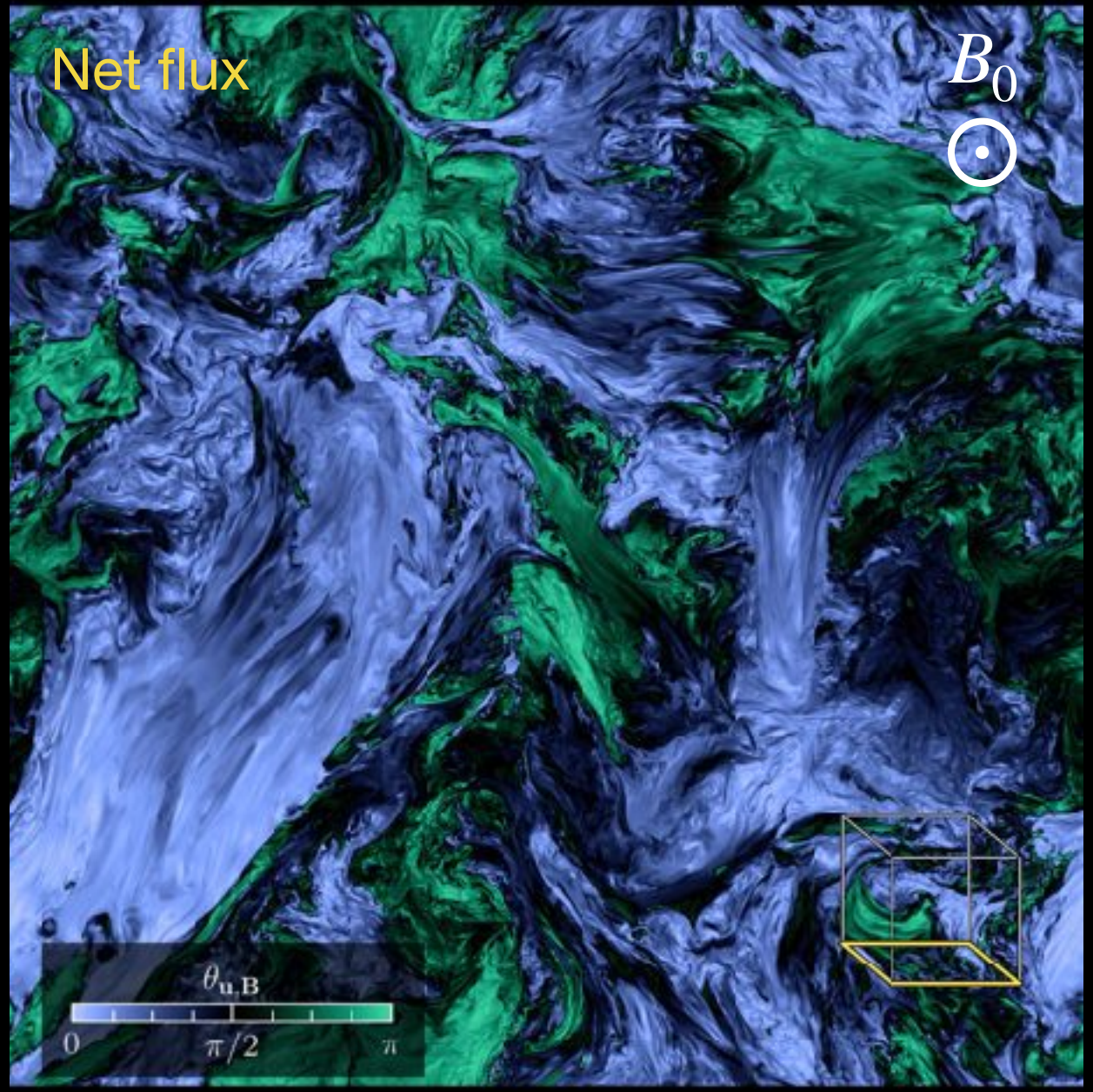
Beattie* & *Bhattacharjee (2026; PRL). Local relaxation and scale-dependent alignment in compressible, magnetized turbulence





Patchy / cellular alignment

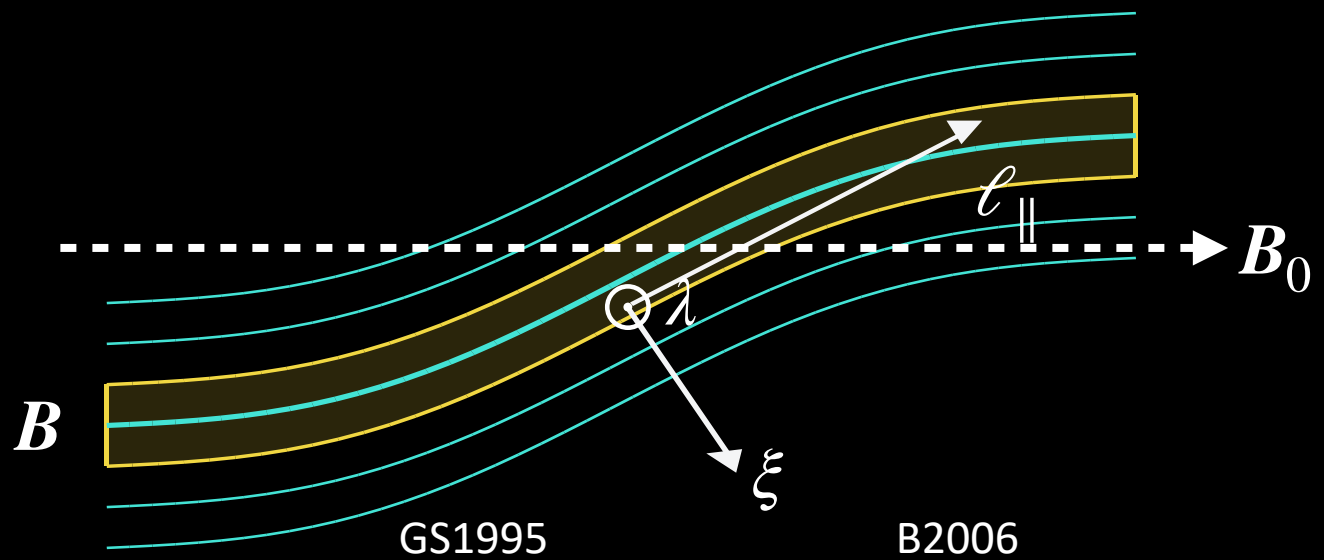


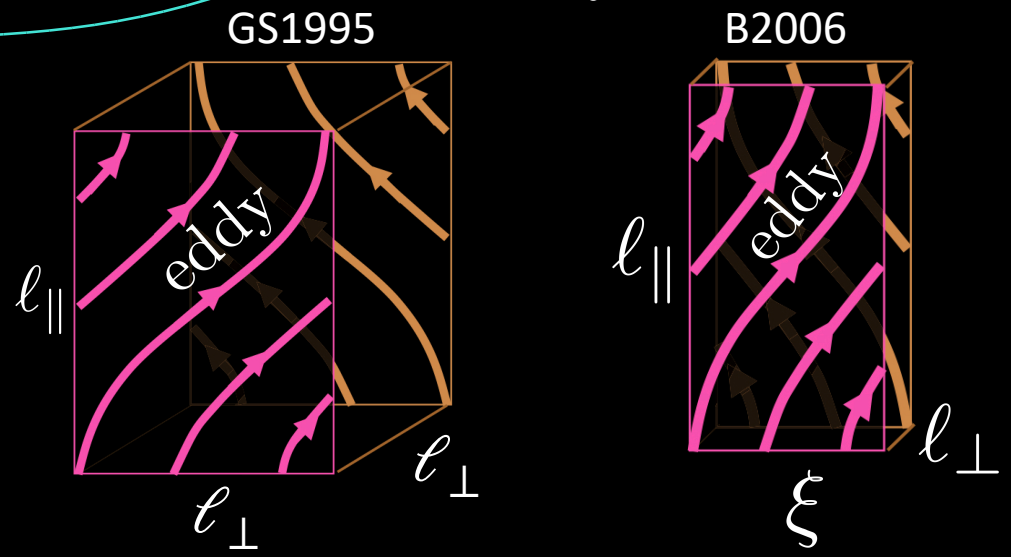


Mason+2006

How does alignment change with scale?

Measure increments in the local eddy frame





$$\delta f = f(x + \ell) - f(x)$$

$$\hat{b}_\ell = \frac{B(x) + B(x + \ell)}{|B(x) + B(x + \ell)|}$$

$$\delta f_\perp = (\mathrm{I} - \hat{b}_\ell \hat{b}_\ell)\, \delta f$$

$$\theta_{f,g}(\lambda) \simeq \frac{\langle |\delta f_\perp \times \delta g_\perp| \rangle_\lambda}{\langle |\delta f_\perp| |\delta g_\perp| \rangle_\lambda}$$

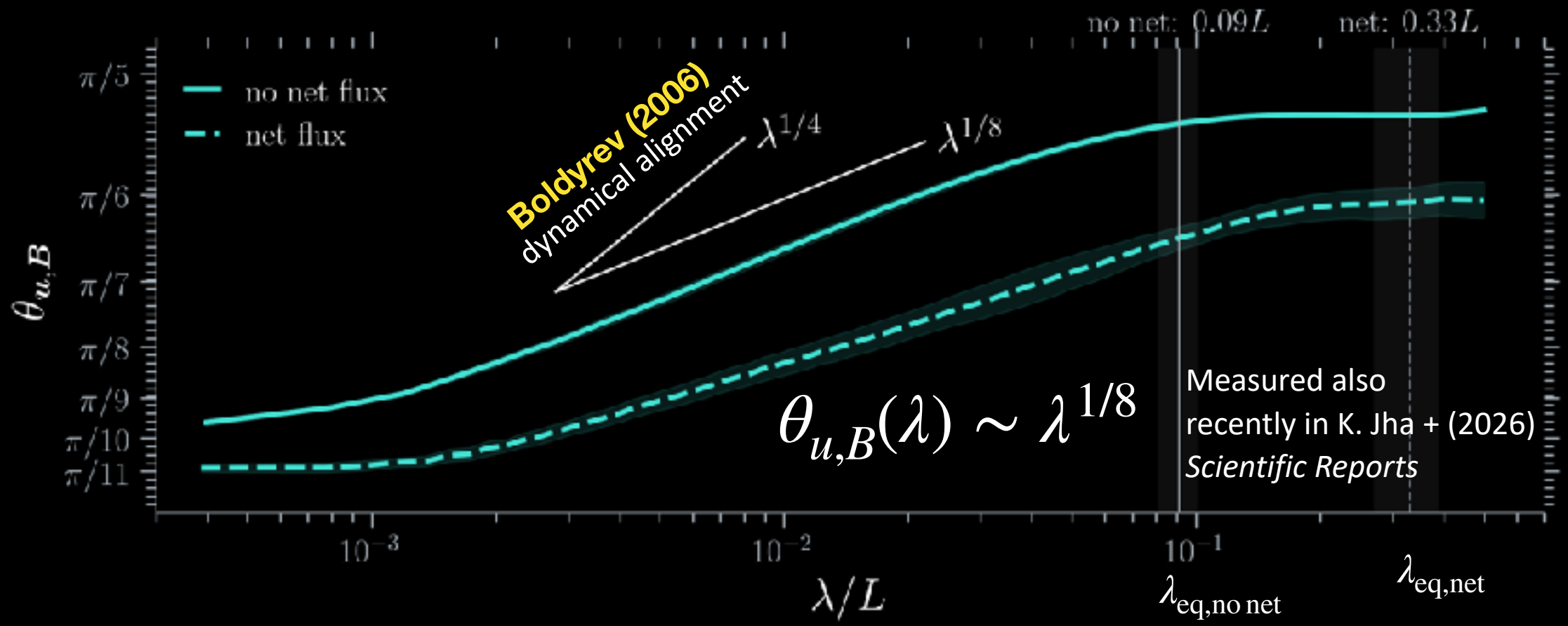
Note:

$$\theta_{u,B}(\lambda) \sim \ell_\perp / \xi$$

Alfvénic alignment: one quarter or one eighth?

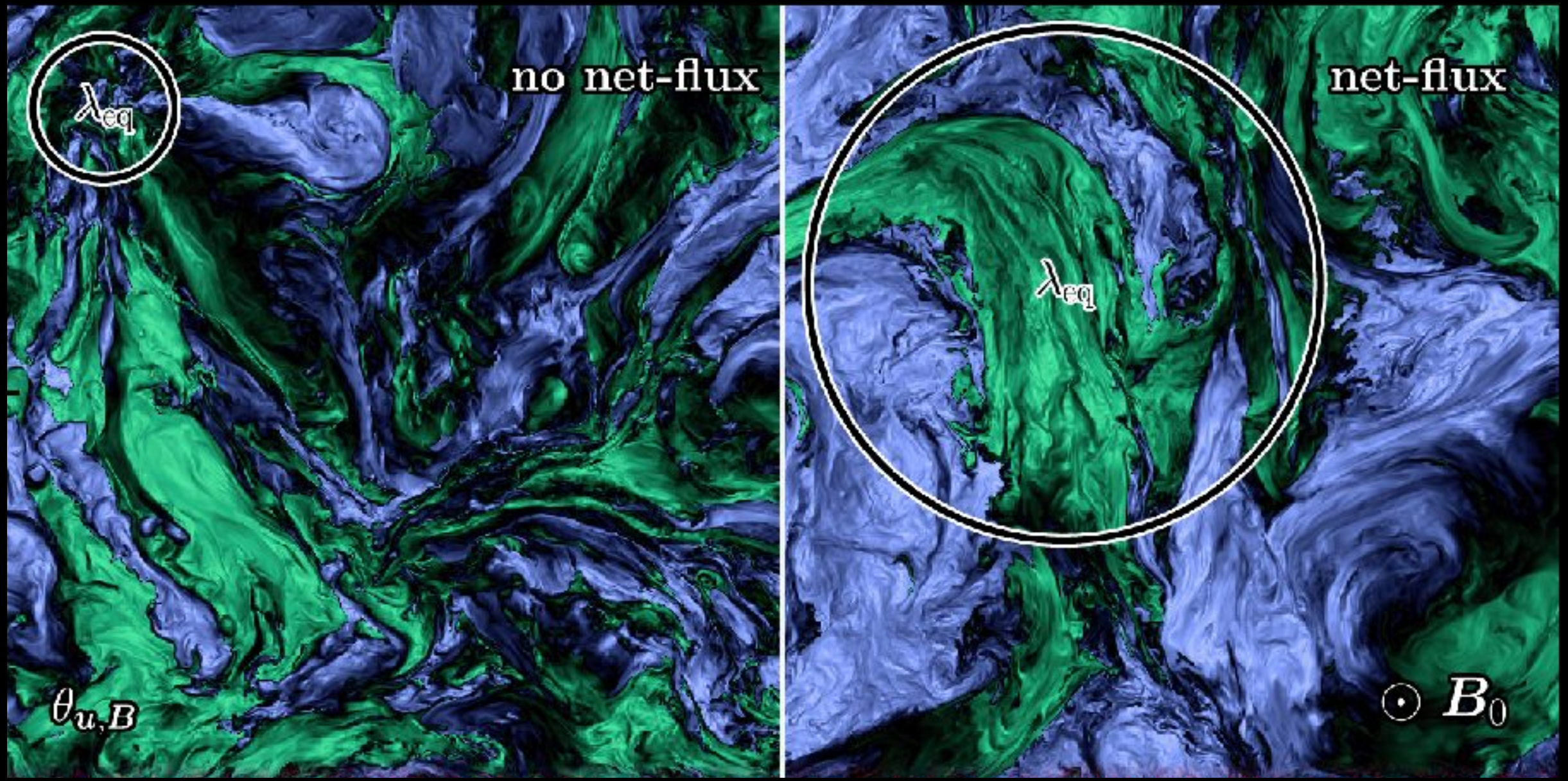
$$E_{\mathrm{kin}}(\lambda_{\mathrm{eq}}) = E_{\mathrm{mag}}(\lambda_{\mathrm{eq}})$$

$$t_A < t_{\mathrm{eddy}}$$



$$t_A > t_{\mathrm{eddy}}$$

Quarter-power prediction: Boldyrev (2006), PRL 96, 115002


no net-flux
λ_{eq}
$\theta_{u,B}$
net-flux
λ_{eq}
$\odot\ B_0$

Small departures from an Alfvénic state below the energy equipartition scale

Nearly equal amplitudes

$$E_{\mathrm{kin}}(\lambda_{\mathrm{eq}}) = E_{\mathrm{mag}}(\lambda_{\mathrm{eq}})$$

A distance to the nearest aligned state
(a criteria for cross-helicity conserving relaxation)

$$\mathscr{D}_A(\lambda) = \min_{\sigma=\pm 1} \frac{|\,\delta\boldsymbol{u}_\lambda - \sigma\delta\boldsymbol{B}_\lambda\,|^2}{|\,\delta\boldsymbol{u}_\lambda\,|^2 + |\,\delta\boldsymbol{B}_\lambda\,|^2} \simeq \frac{1}{2}\theta^2_{\boldsymbol{u},\boldsymbol{B}}$$

small angles

Crank the constant flux argument

$$\varepsilon_{D_A} \sim \frac{\delta u^2_{\ell_\perp}\mathscr{D}_A}{t_{\mathrm{nl}}} \simeq \mathrm{const}\,. \qquad \delta u_{\ell_\perp} \sim \ell_\perp^{1/4} \implies \theta_{u,B} \sim \ell_\perp^{1/8}$$

Small departures from an Alfvénic state below the energy equipartition scale

Nearly equal amplitudes

$$E_{\rm kin}(\lambda_{\rm eq}) = E_{\rm mag}(\lambda_{\rm eq})$$

A distance to the nearest aligned state

$$\mathscr{D}_A(\lambda) = \min_{\sigma=\pm 1} \frac{|\,\delta \boldsymbol{u}_\lambda - \sigma \delta \boldsymbol{B}_\lambda\,|^2}{|\,\delta \boldsymbol{u}_\lambda\,|^2 + |\,\delta \boldsymbol{B}_\lambda\,|^2} \simeq \frac{1}{2}\theta^2_{\boldsymbol{u},\boldsymbol{B}}$$

small angles

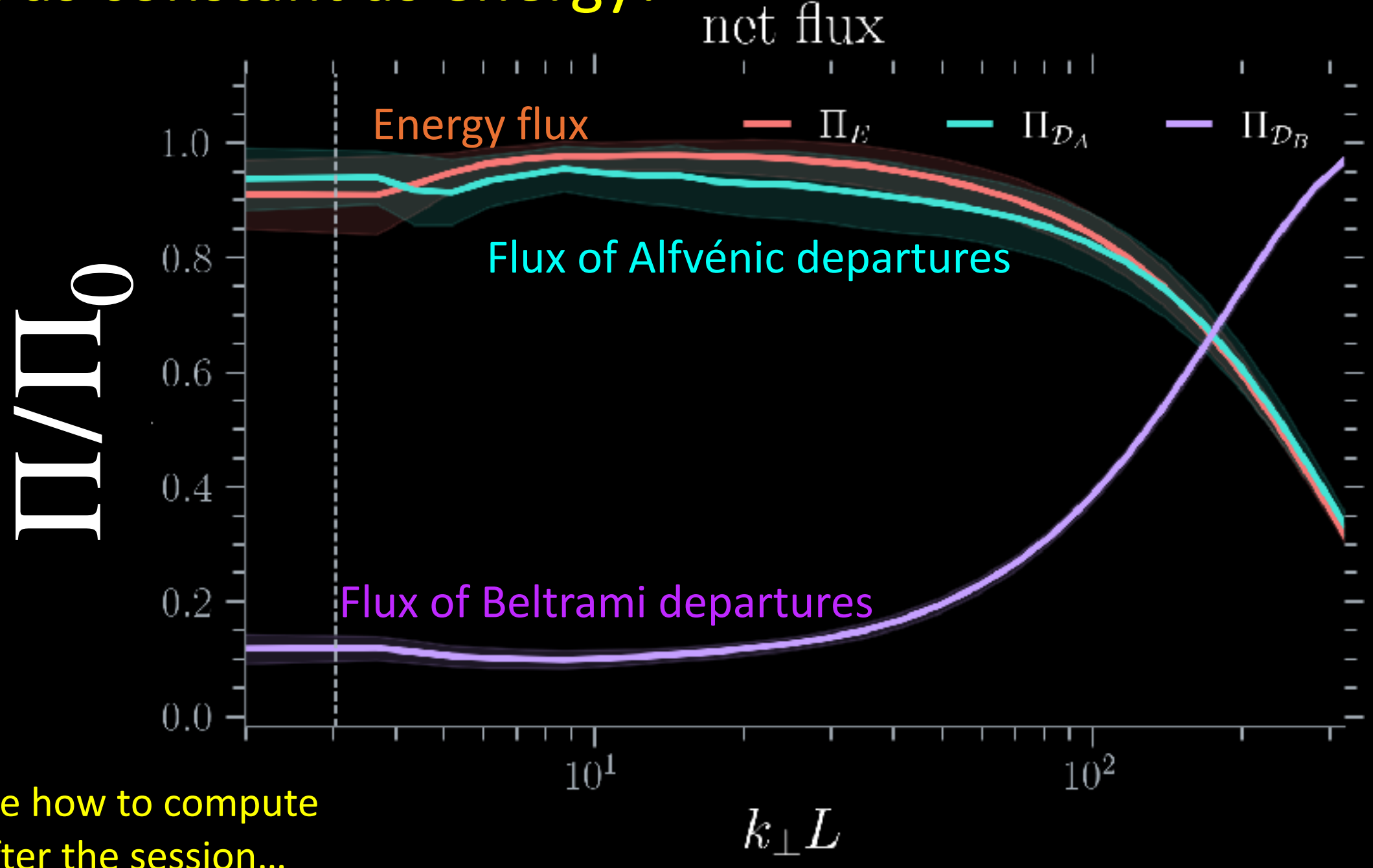
Let's test!!!

Crank the constant flux argument

$$\varepsilon_{D_A} \sim \frac{\delta u^2_{\ell_\perp} \mathscr{D}_A}{t_{\rm nl}} \simeq \text{const}\,.$$

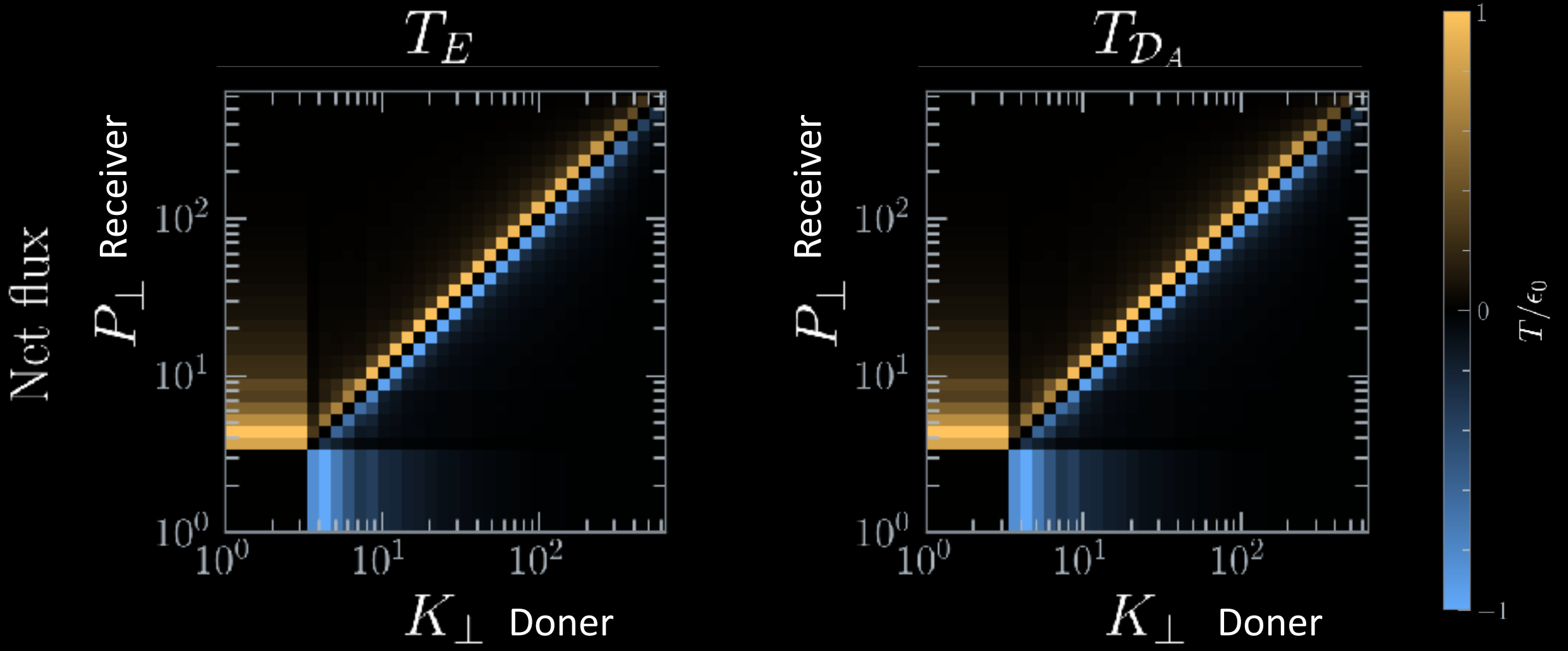
$$\delta u_{\ell_\perp} \sim \ell_\perp^{1/4} \quad \Longrightarrow \quad \theta_{u,B} \sim \ell_\perp^{1/8}$$

Just as constant as energy!



Ask me how to compute this after the session…

Flux transfers are dominated by local k space interactions (i.e., energy and Alfvénic departures undergo cascade)



Small departures from an Alfvénic state below the energy equipartition scale

Nearly equal amplitudes

$$E_{\rm kin}(\lambda_{\rm eq}) = E_{\rm mag}(\lambda_{\rm eq})$$

A distance to the nearest aligned state

$$\mathscr{D}_A(\lambda) = \min_{\sigma=\pm 1} \frac{|\,\delta \boldsymbol{u}_\lambda - \sigma \delta \boldsymbol{B}_\lambda\,|^2}{|\,\delta \boldsymbol{u}_\lambda\,|^2 + |\,\delta \boldsymbol{B}_\lambda\,|^2} \simeq \frac{1}{2}\theta^2_{\boldsymbol{u},\boldsymbol{B}}$$

small angles

Let's test!!!

Crank the constant flux argument

$$\varepsilon_{D_A} \sim \frac{\delta u^2_{\ell_\perp} \mathscr{D}_A}{t_{\rm nl}} \simeq \text{const}\,.$$



$$\delta u_{\ell_\perp} \sim \ell_\perp^{1/4} \quad \Longrightarrow \quad \theta_{u,B} \sim \ell_\perp^{1/8}$$

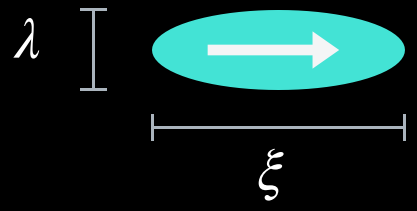
Weaker alignment raises the reconnection-mediated threshold

Eddy cross-section

B2006 alignment $\theta_\lambda \sim \lambda^{1/4}$



BB2026 alignment $\theta_\lambda \sim \lambda^{1/8}$



Conditional critical magnetic Reynolds number $t_{\rm nl} \sim (\gamma_{\rm tearing})^{-1}$

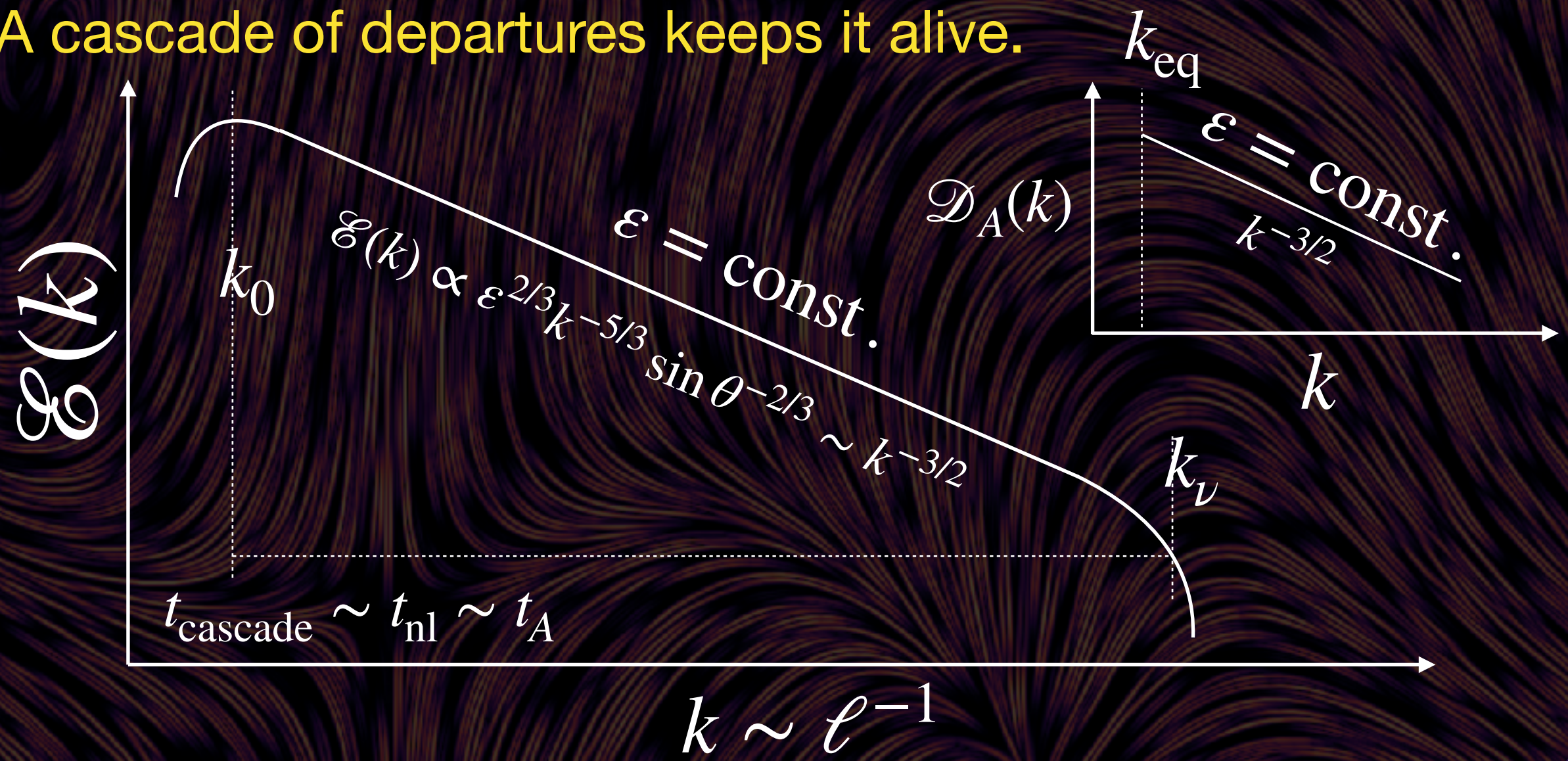
$$\mathrm{Rm}_c \sim 10^5 \longrightarrow \mathcal{O}(10^8)$$

$$k_* L : \quad \mathrm{Rm}^{4/7} \longrightarrow \mathrm{Rm}^{2/3}$$

$$\mathrm{Rm}_{c,\,\mathrm{new}} \sim (\mathrm{Rm}_{c,\,\mathrm{old}})^{12/7}$$

Same tearing / resistive scale ratio

Summary. MHD turbulence becomes aligned. A cascade of departures keeps it alive.



Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).