The turbulent dynamo across space and time: the chronology of high-k modes

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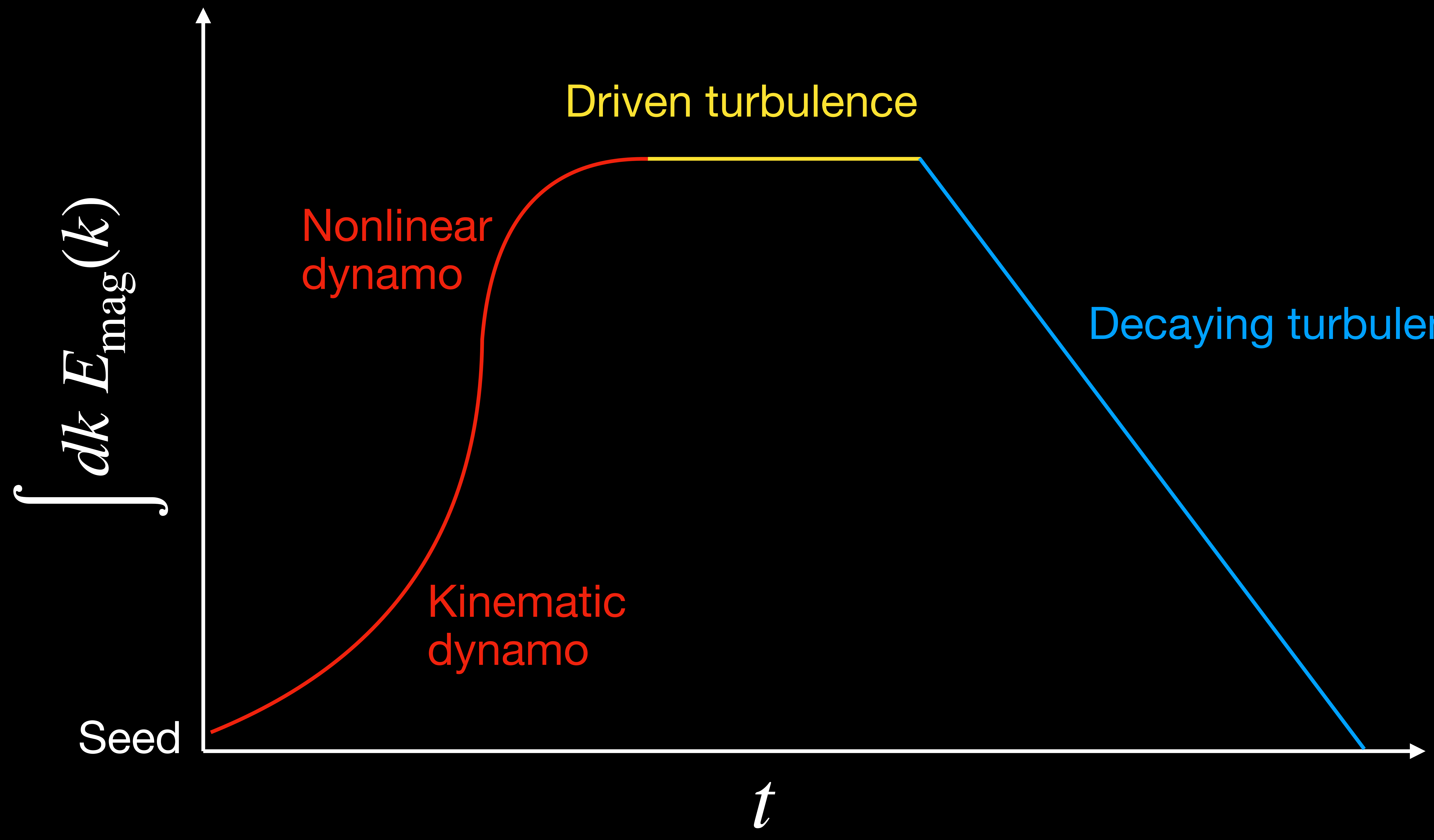
Collaborators for this work: Neco Kriel (UBonn), Amitava Bhattacharjee (PU), Christoph Federrath (ANU), Ralf Klessen (UH), Mark Krumholz (ANU), Salvatore Cielo (LRZ)

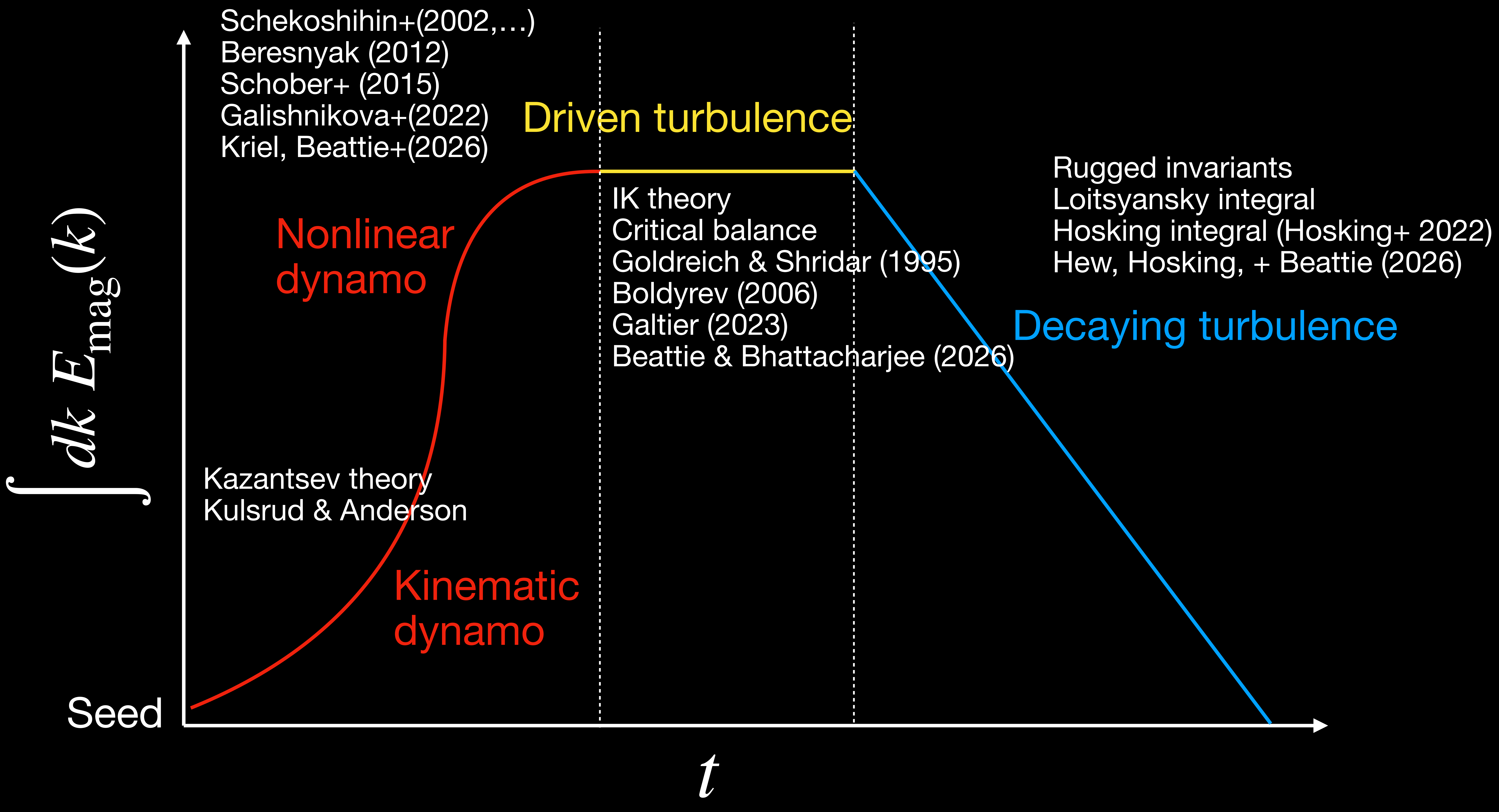
Goals for this afternoon seminar

1. Get folks excited about the universality of turbulent dynamos
2. Grow a small-scale (kinematic + nonlinear) field in the Dartmouth College seminar.
3. Venture into the dynamo saturation with $10{,}080^3$ domain that can resolve inertial ranges, presumably in the MHD asymptotic state, focussing on the role of alignment.

The lifecycle of a high-k mode spans across a patchwork quilt of theories



The lifecycle of a high-k mode spans across a patchwork quilt of theories



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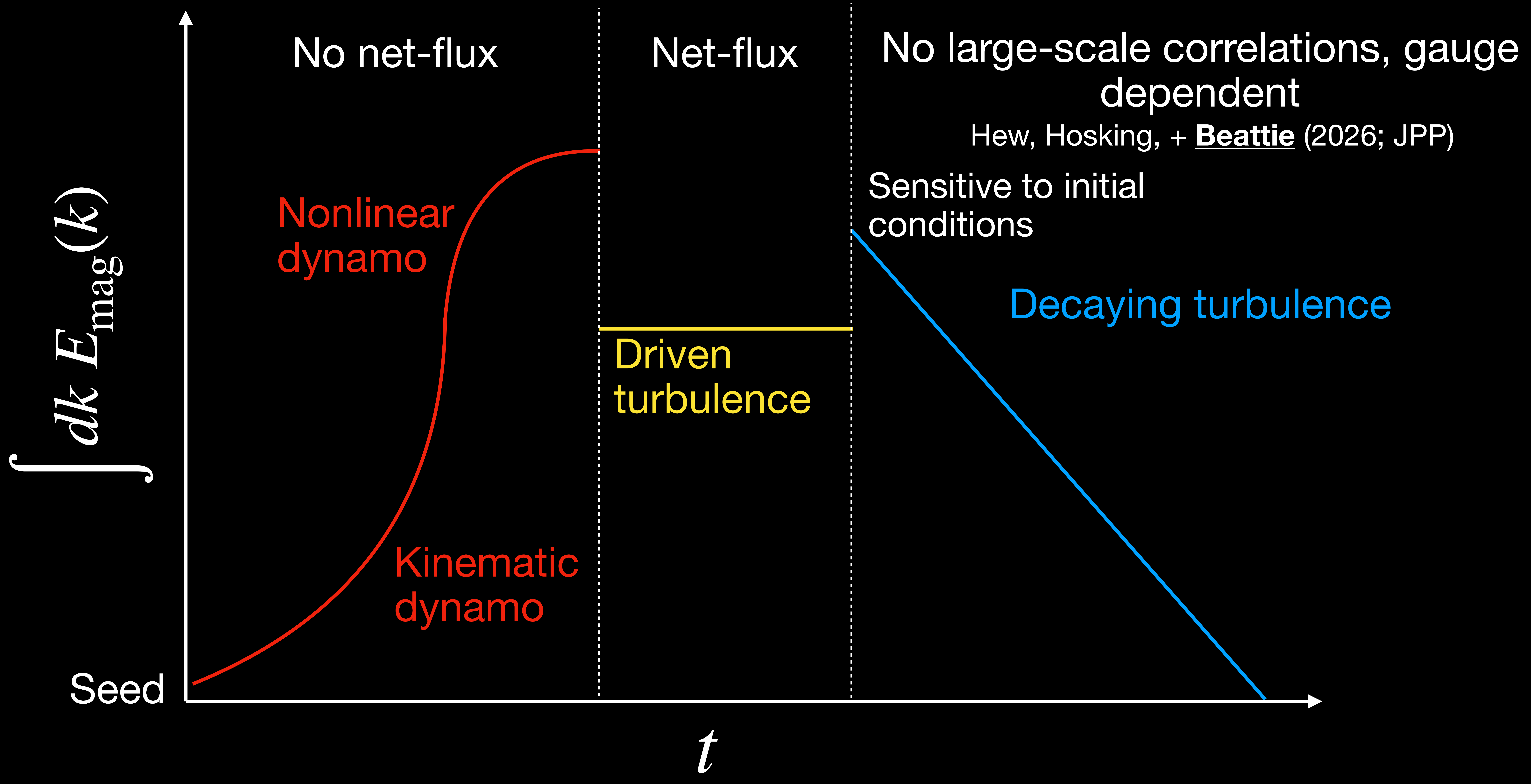
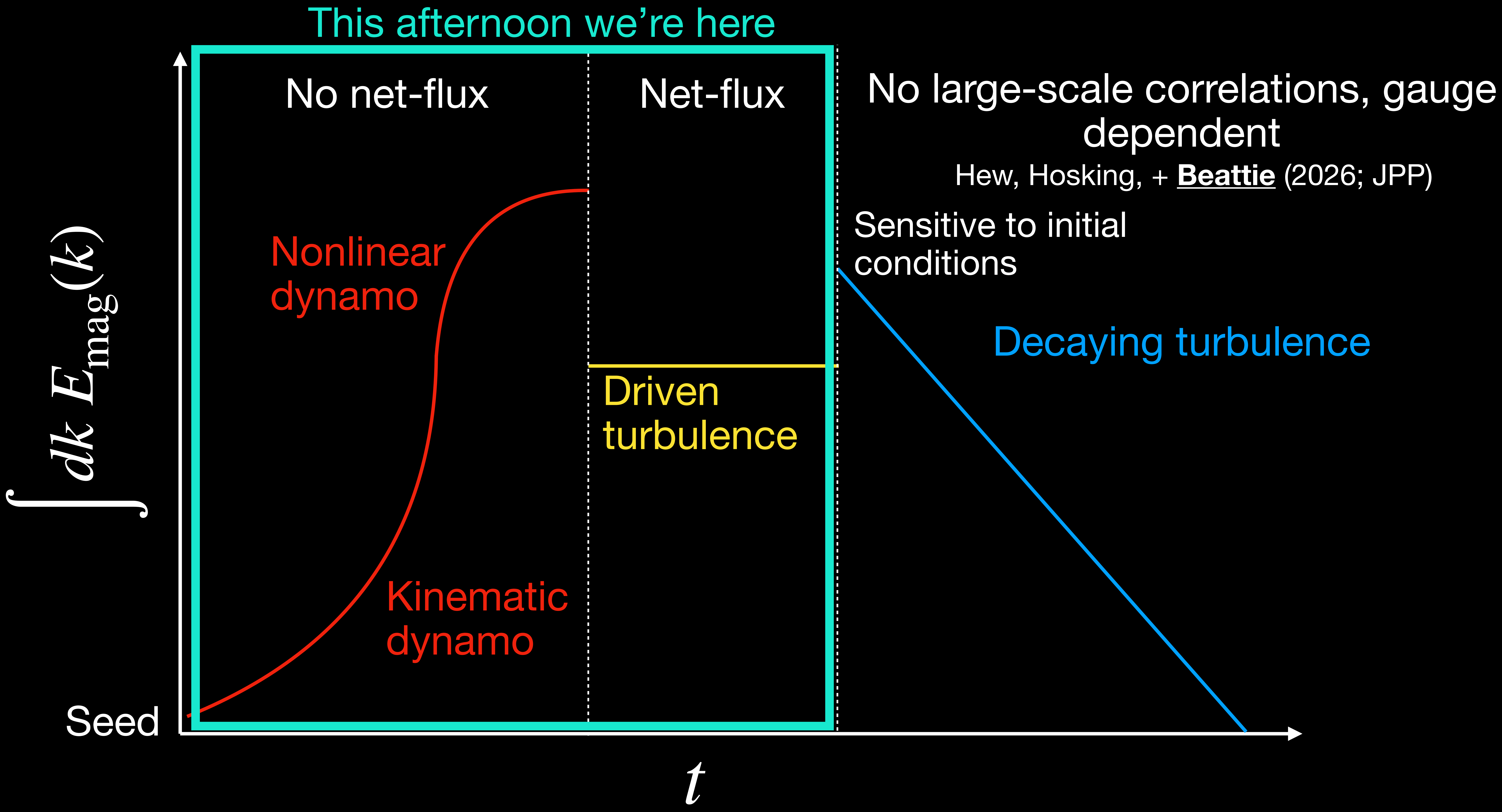
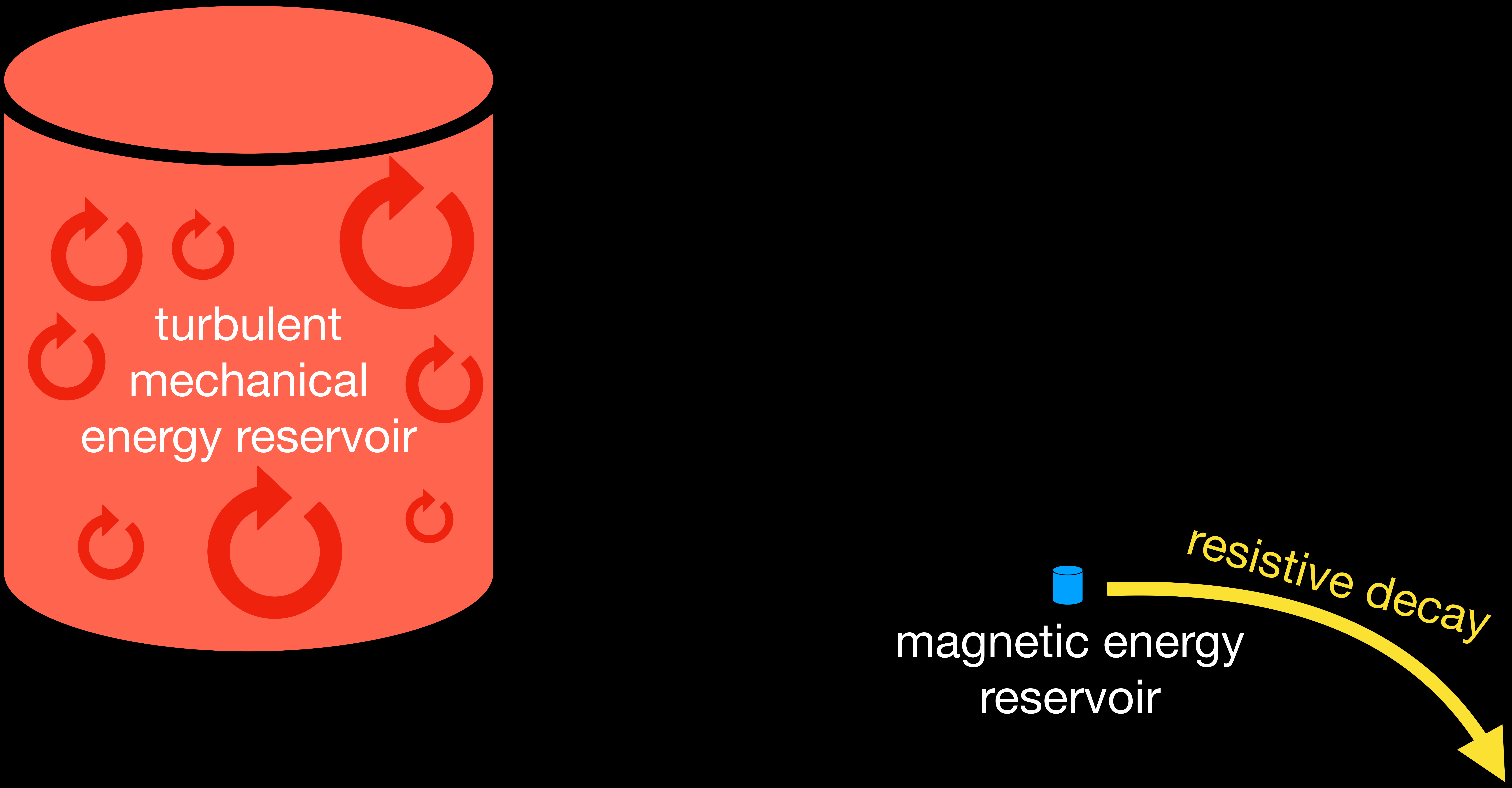


The lifecycle of a high-k mode spans across a patchwork quilt of theories



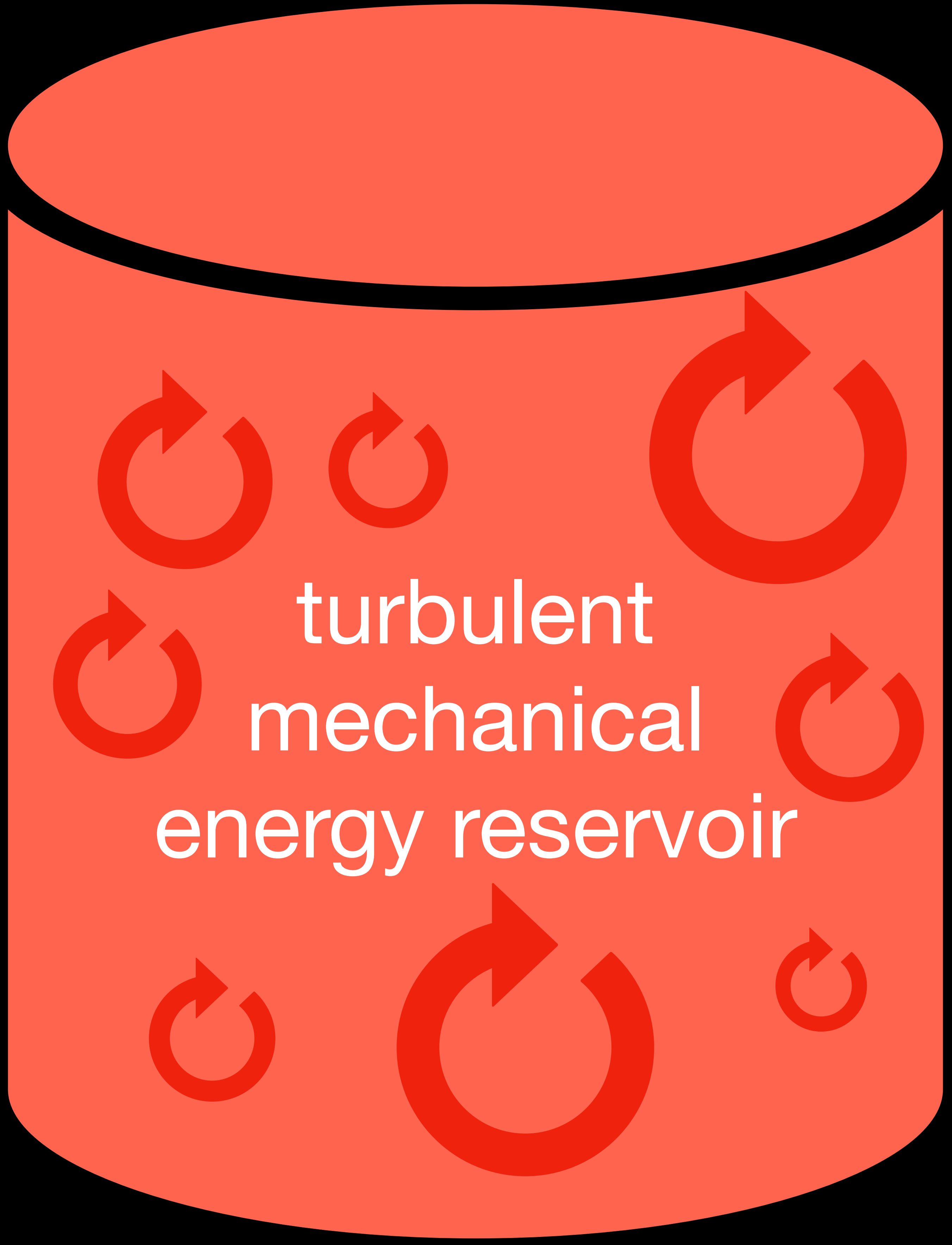
What is a turbulence, magnetic dynamo?

Starting with a seed magnetic field



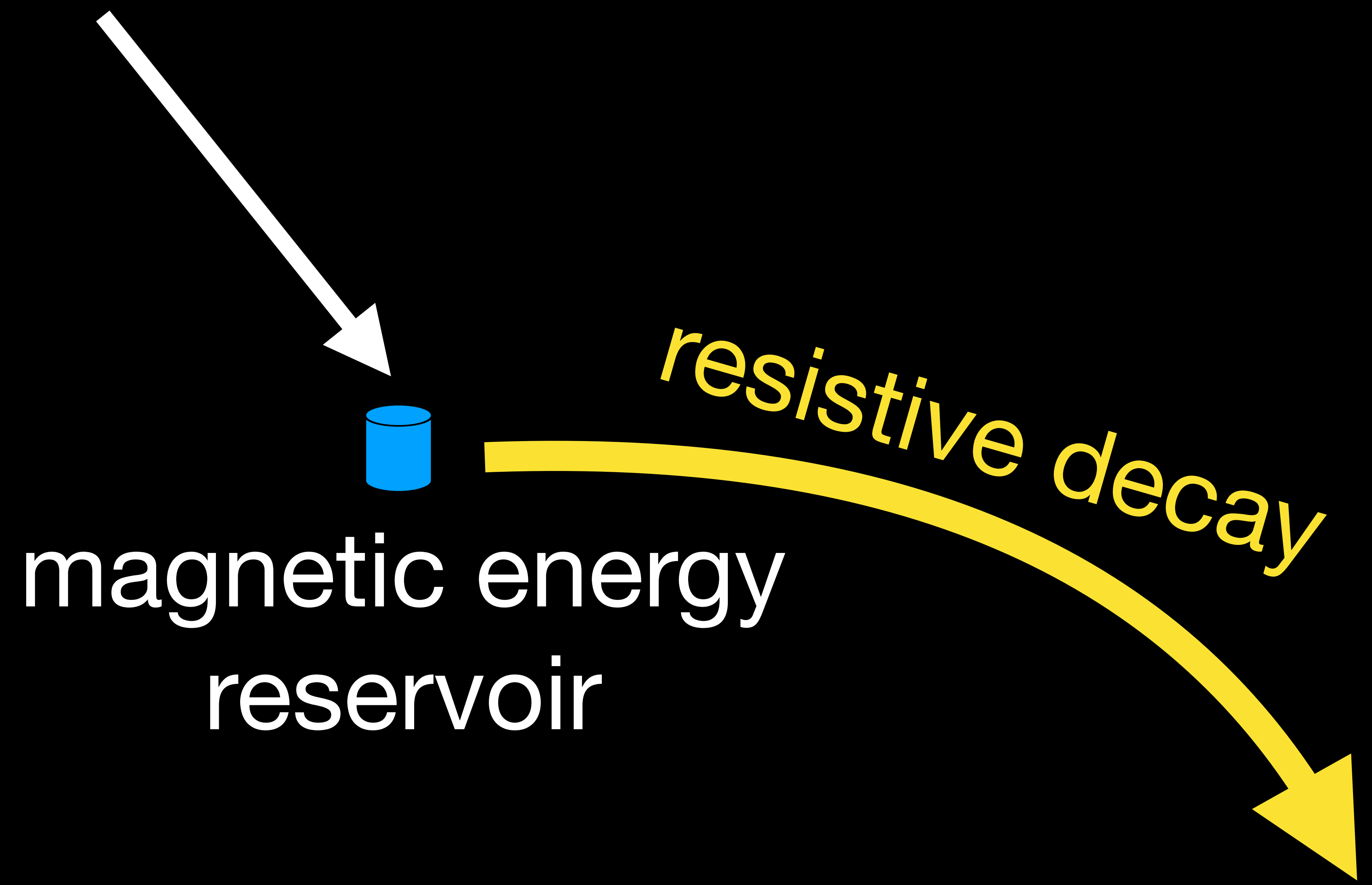
What is a turbulence, magnetic dynamo?

Starting with a seed magnetic field



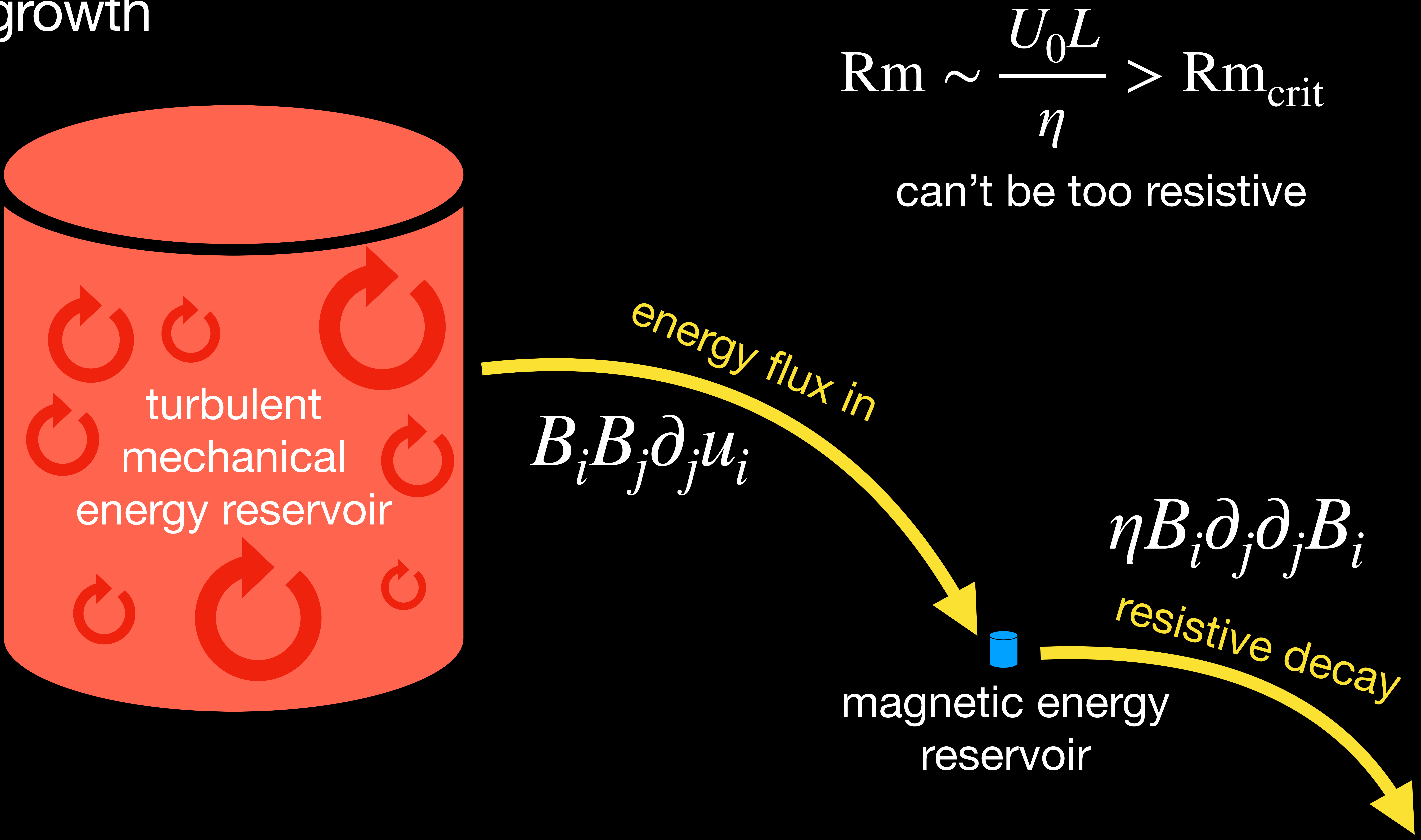
Weibel instability,
Biermann battery,
…
come to IAS and discuss ;)

(Zhou + 2023, 2024, etc.)



What is a turbulence, magnetic dynamo?

Kinematic growth

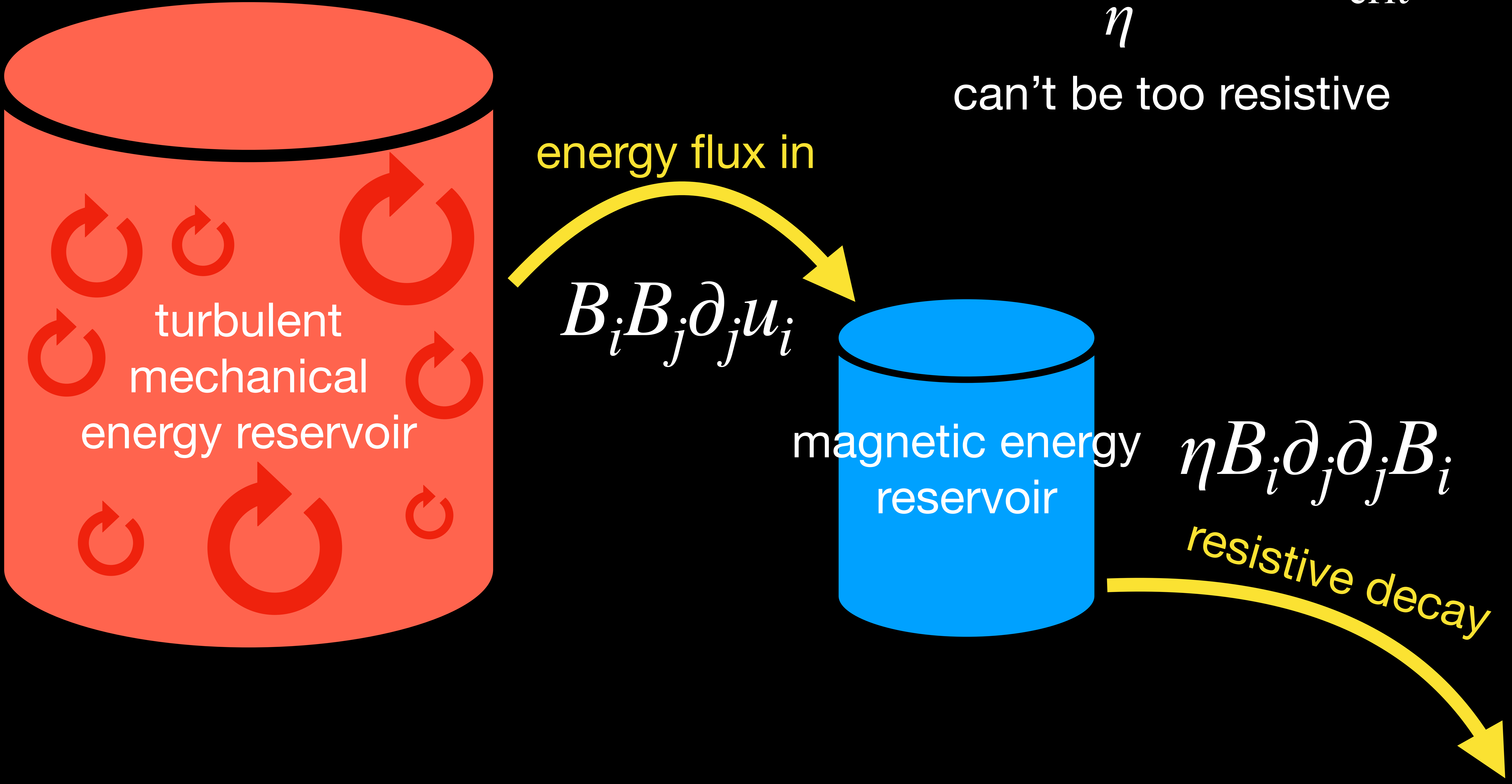


What is a turbulence, magnetic dynamo?

Kinematic growth

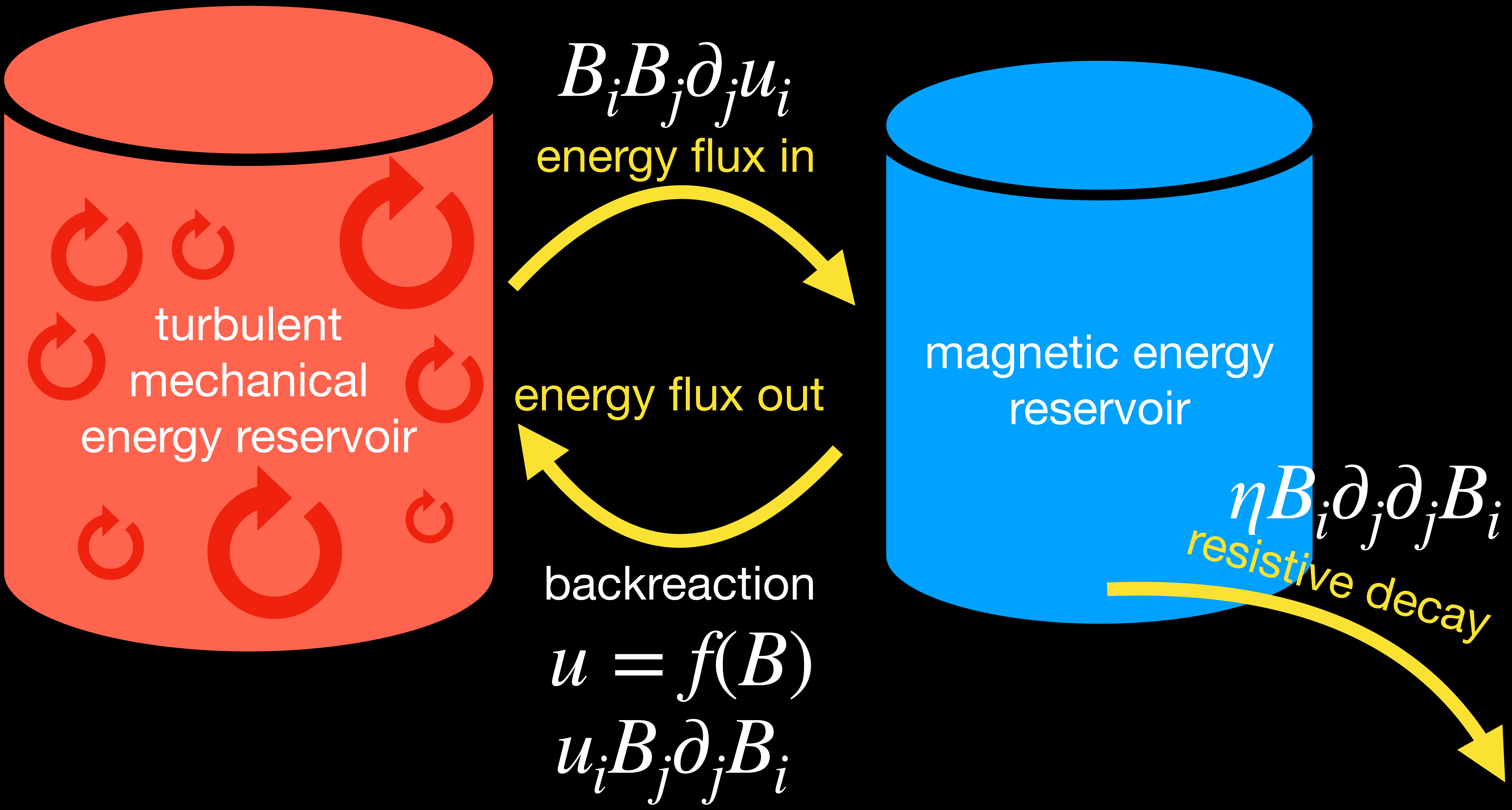
$$\mathrm{Rm} \sim \frac{U_0 L}{\eta} > \mathrm{Rm}_{\mathrm{crit}}$$

can't be too resistive



What is a turbulence, magnetic dynamo?

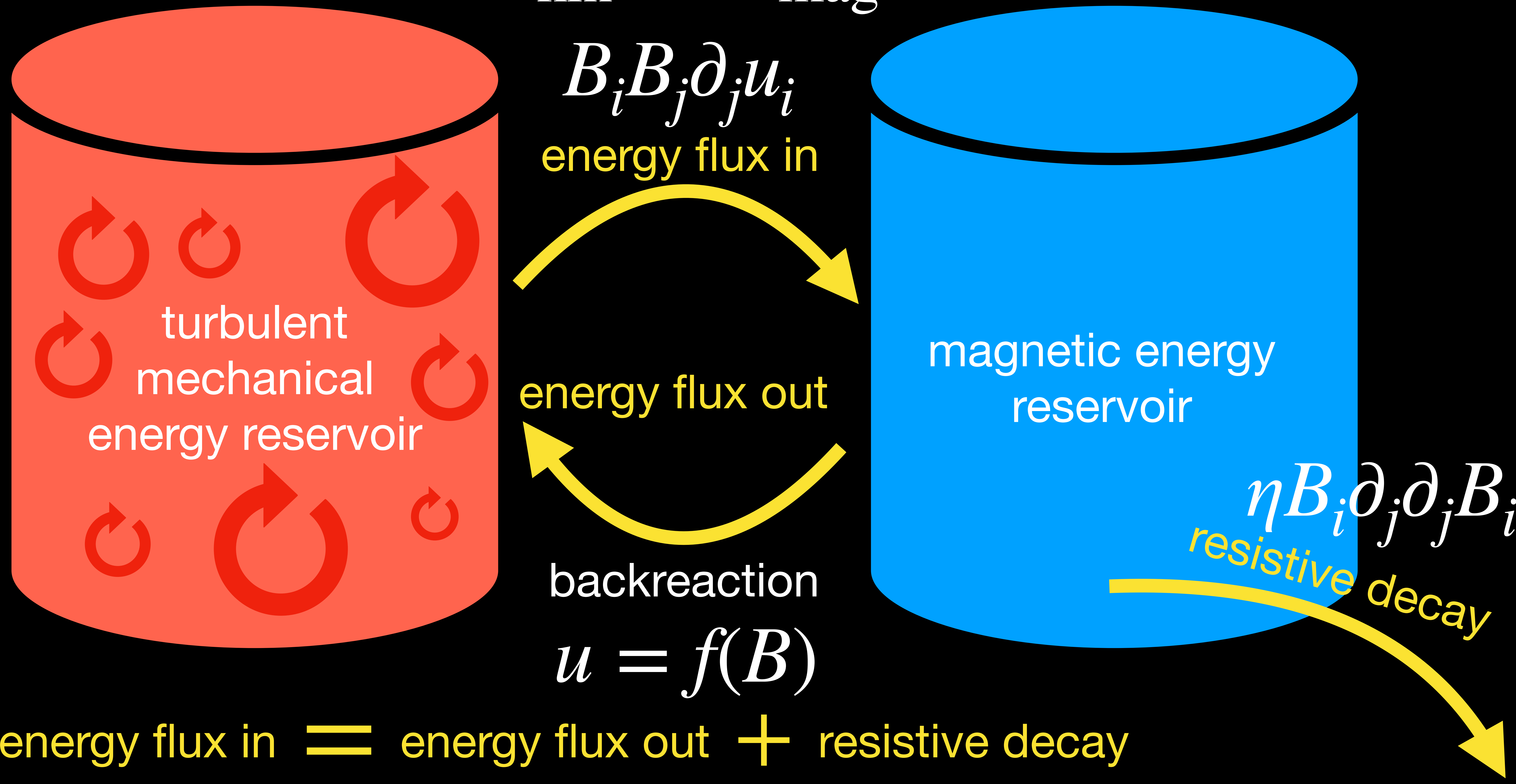
Nonlinearities and backreaction



What is a turbulence, magnetic dynamo?

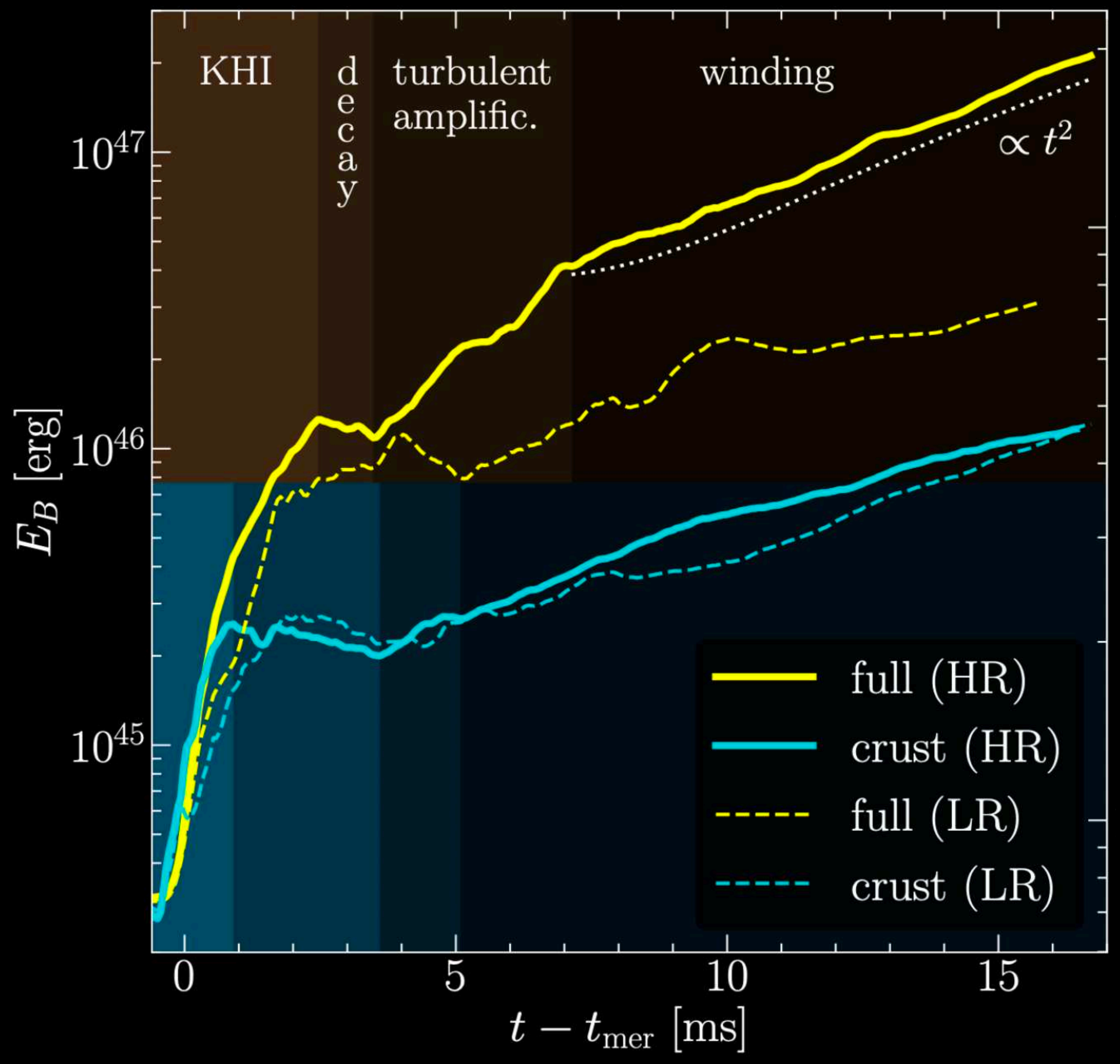
Saturation

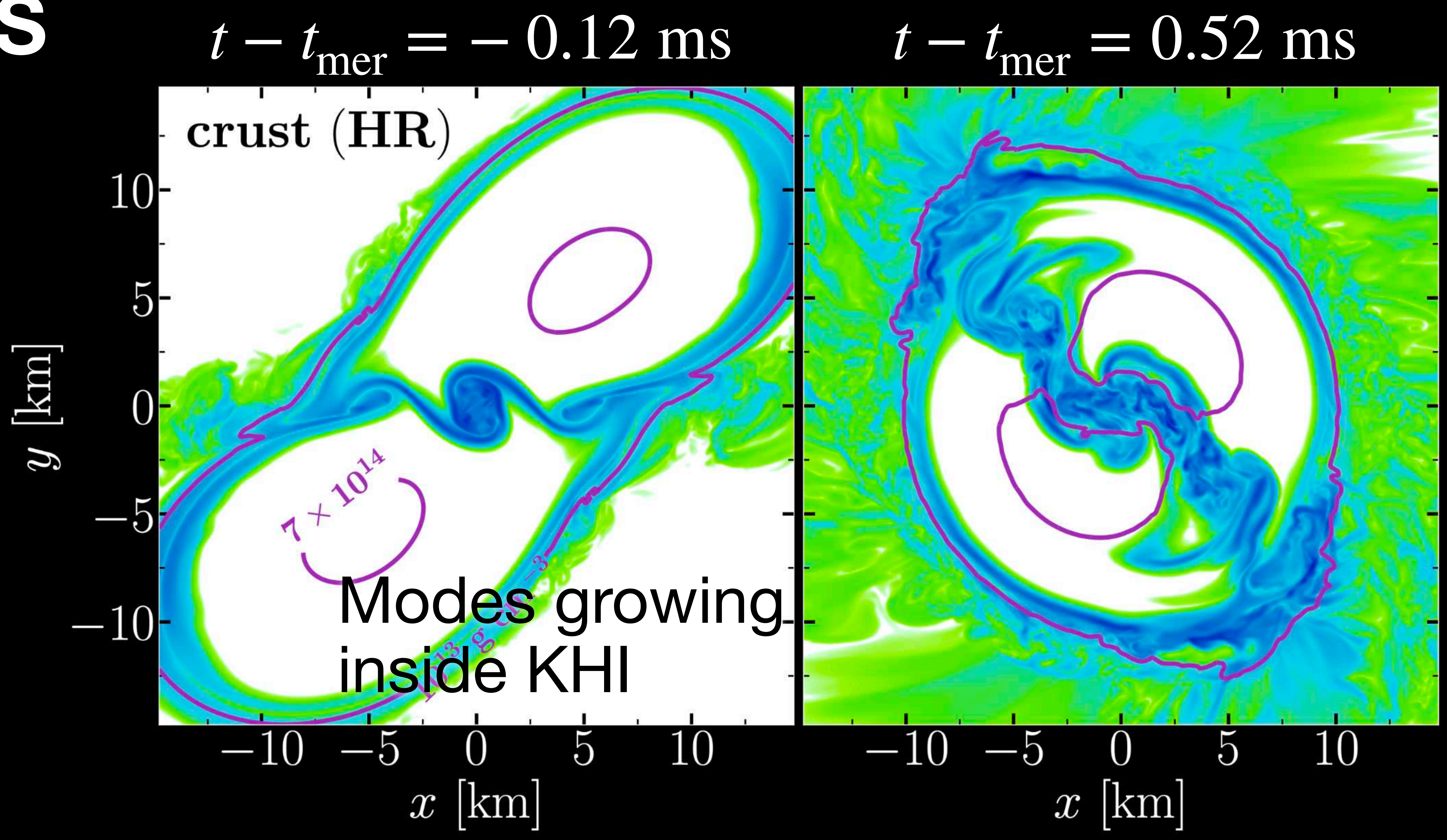
$$\mathcal{E}_{\rm kin} \sim \mathcal{E}_{\rm mag}$$



Examples of turbulent dynamos

KHI instabilities in merging NS





exponential growth → pause

exponential growth ← → LSD

ILES
FIL

Chabanov+ (2023)

Examples of turbulent dynamos

Milky Way-type galaxies in cosmological sims

exponential growth

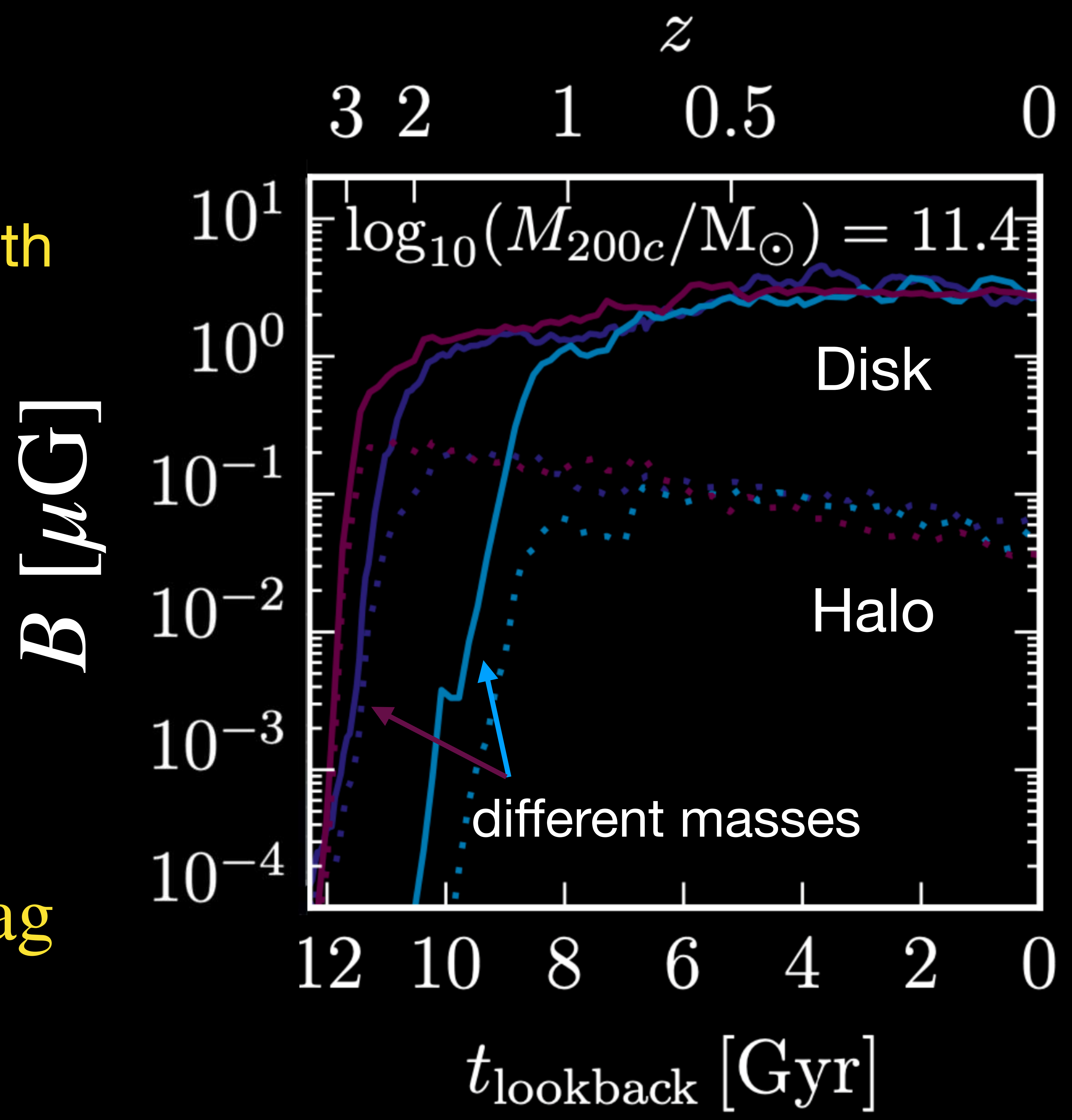
↓

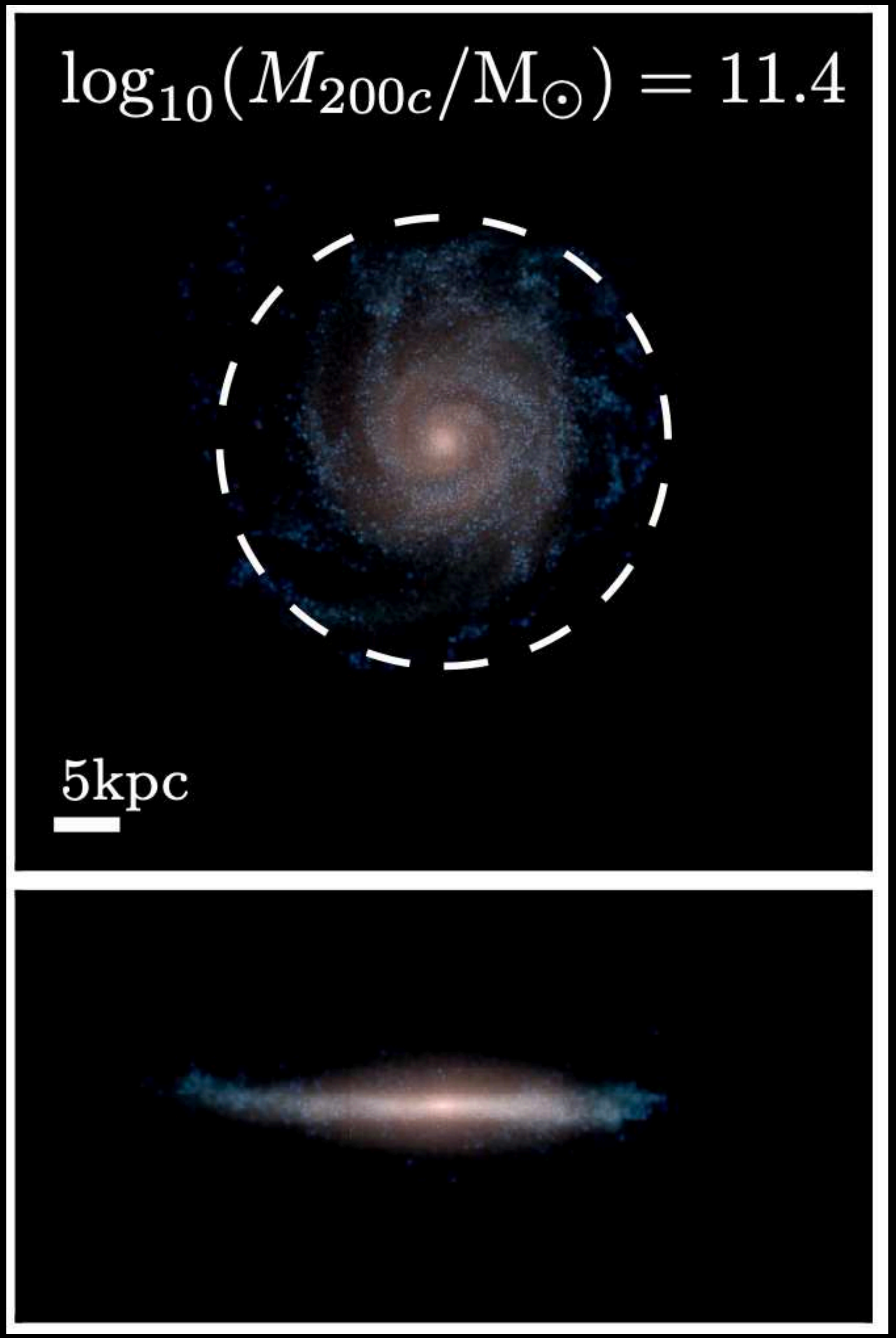
linear growth

↓

saturation

$\mathcal{E}_{\rm kin} \sim \mathcal{E}_{\rm mag}$



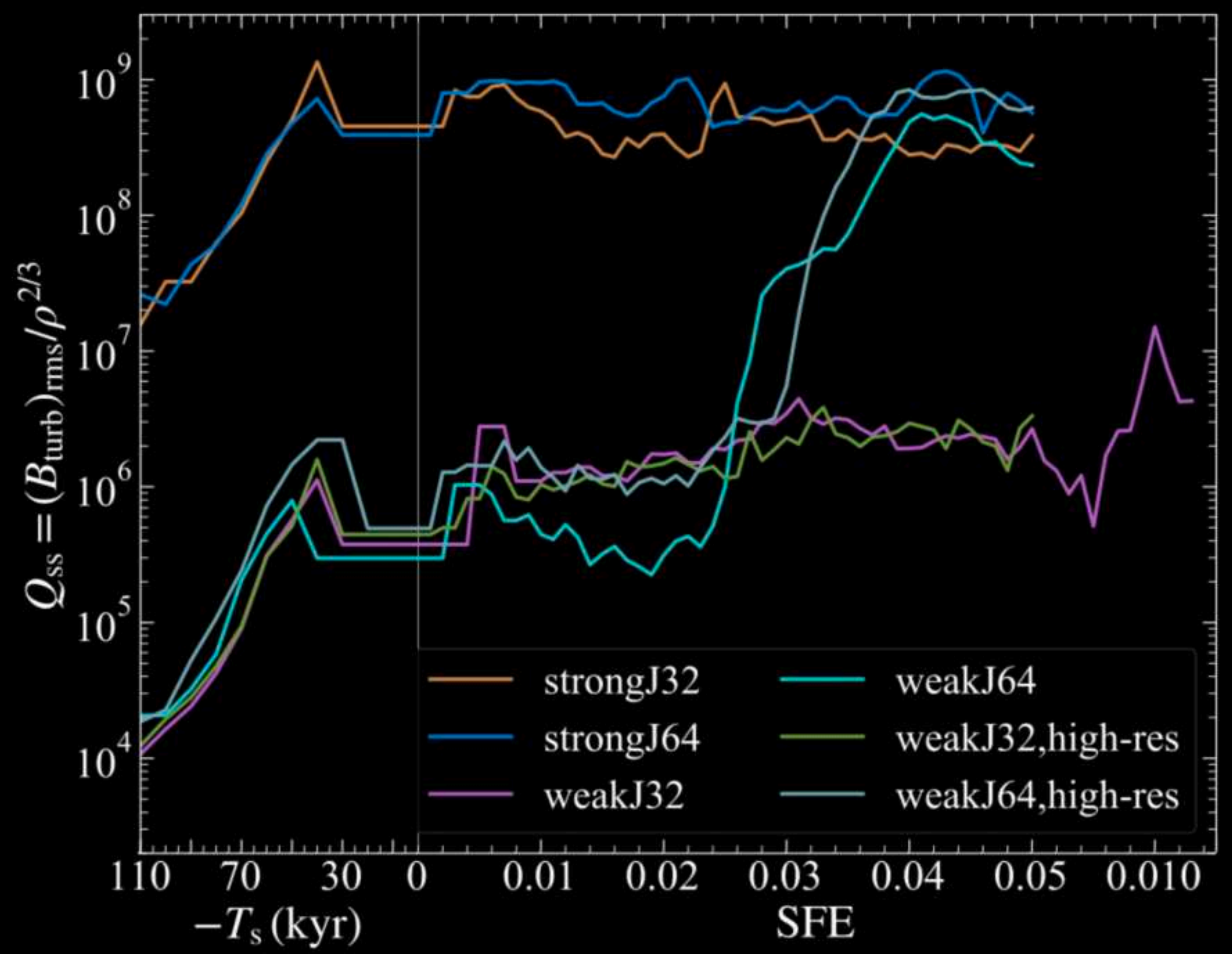


Pakmor+ (2024) *AREPO*

Examples of turbulent dynamos

There are many, across all scales (all MHD)

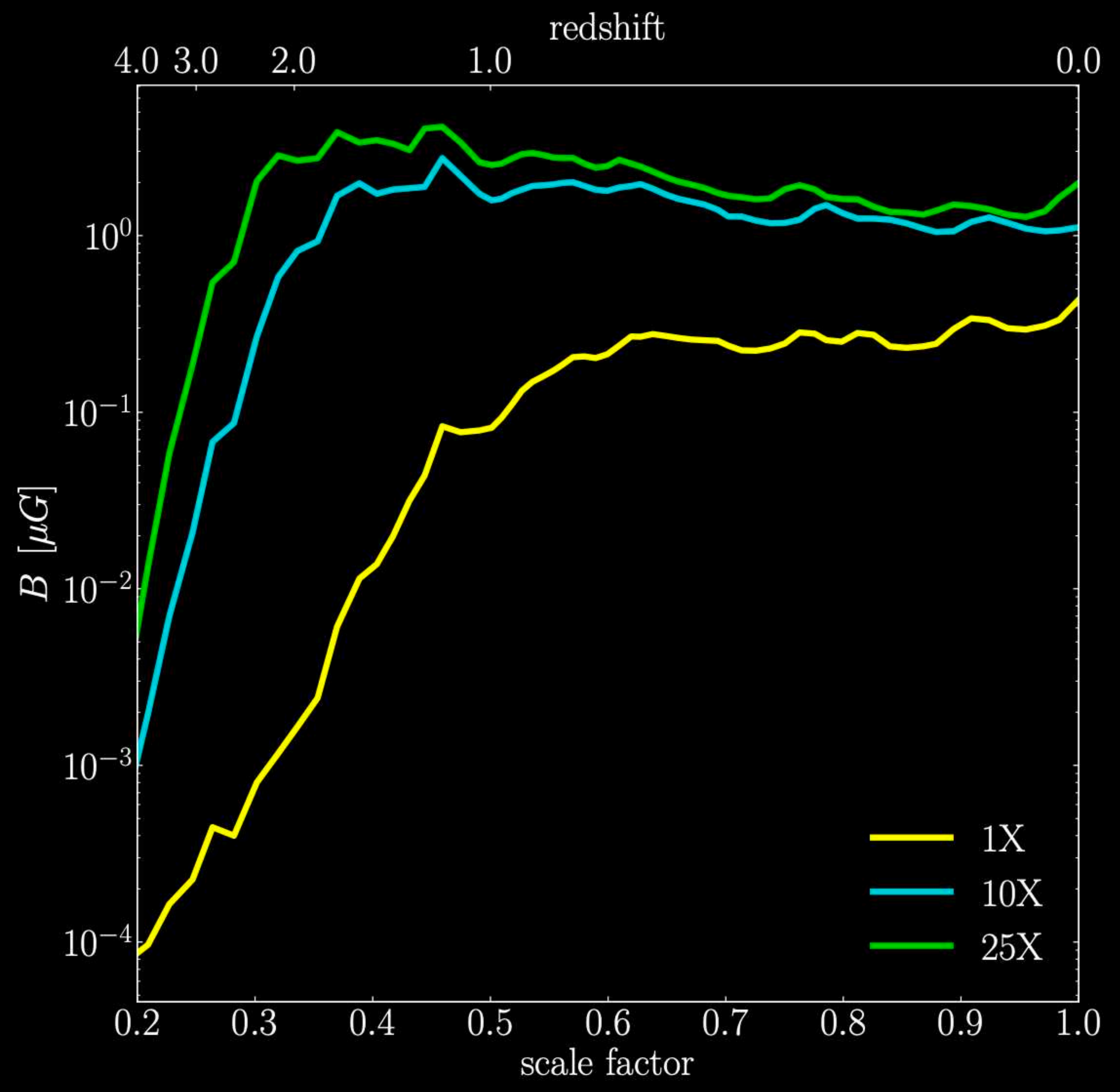
Sharda+2021



Molecular clouds in first generation stars

FLASH

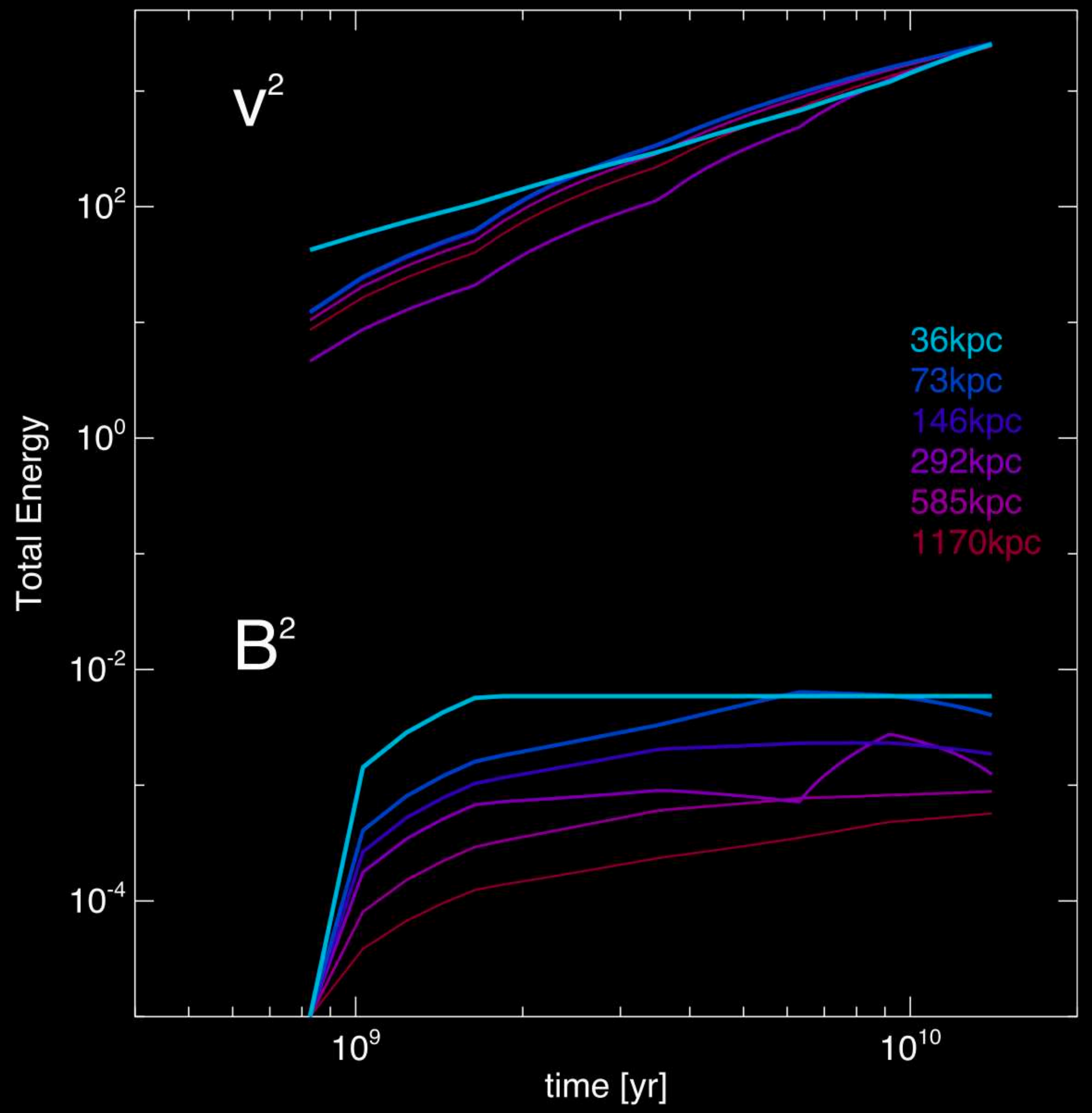
Steinwandel+2021



Intracluster medium

RAMSES

Vazza+2014



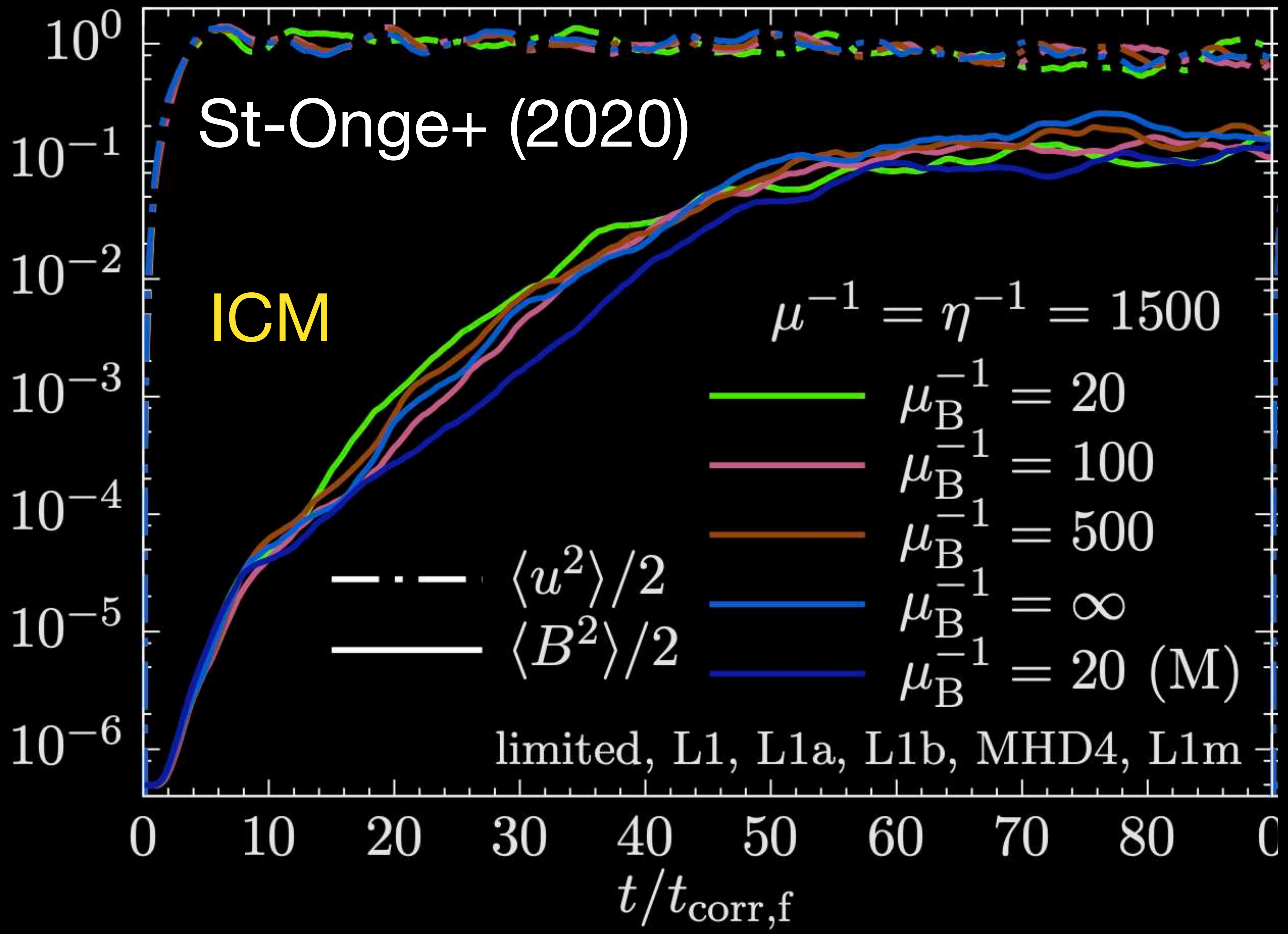
Cosmic filaments

ENZO

Examples of turbulence dynamos

and plasma regimes

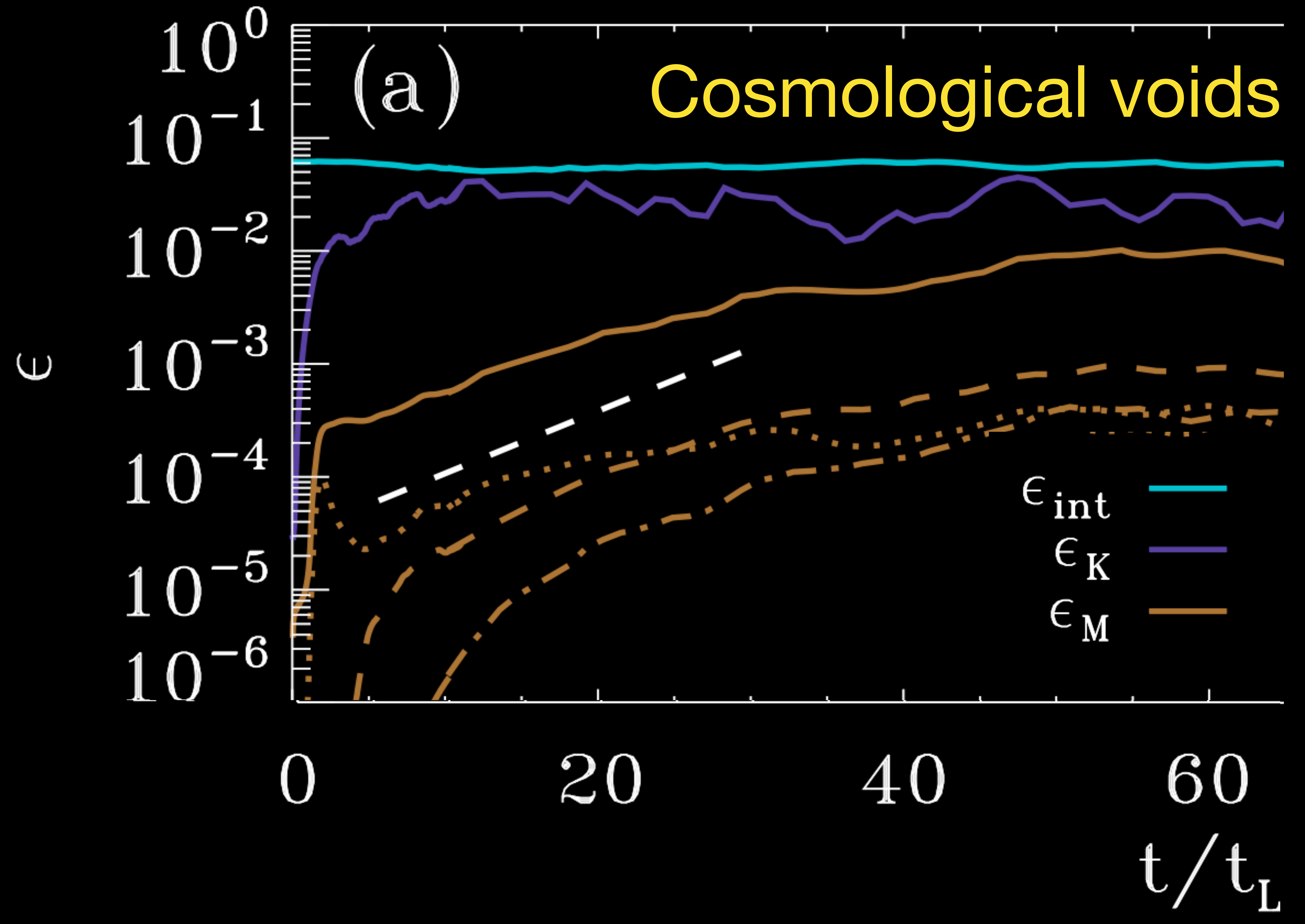
Weakly collisional Braginksii MHD



(added anisotropic viscous Braginskii stress term into MHD)

$\nabla \cdot (\hat{\mathbf{b}} \otimes \hat{\mathbf{b}}(\hat{\mathbf{b}} \otimes \hat{\mathbf{b}} : \nabla \mathbf{v}))$ *Snoopy*

Collisionless plasma magnetogenesis coupled to dynamo



Sironi+2023

(PIC: pair plasma)

TRISTAN—MP

Turbulence dynamos

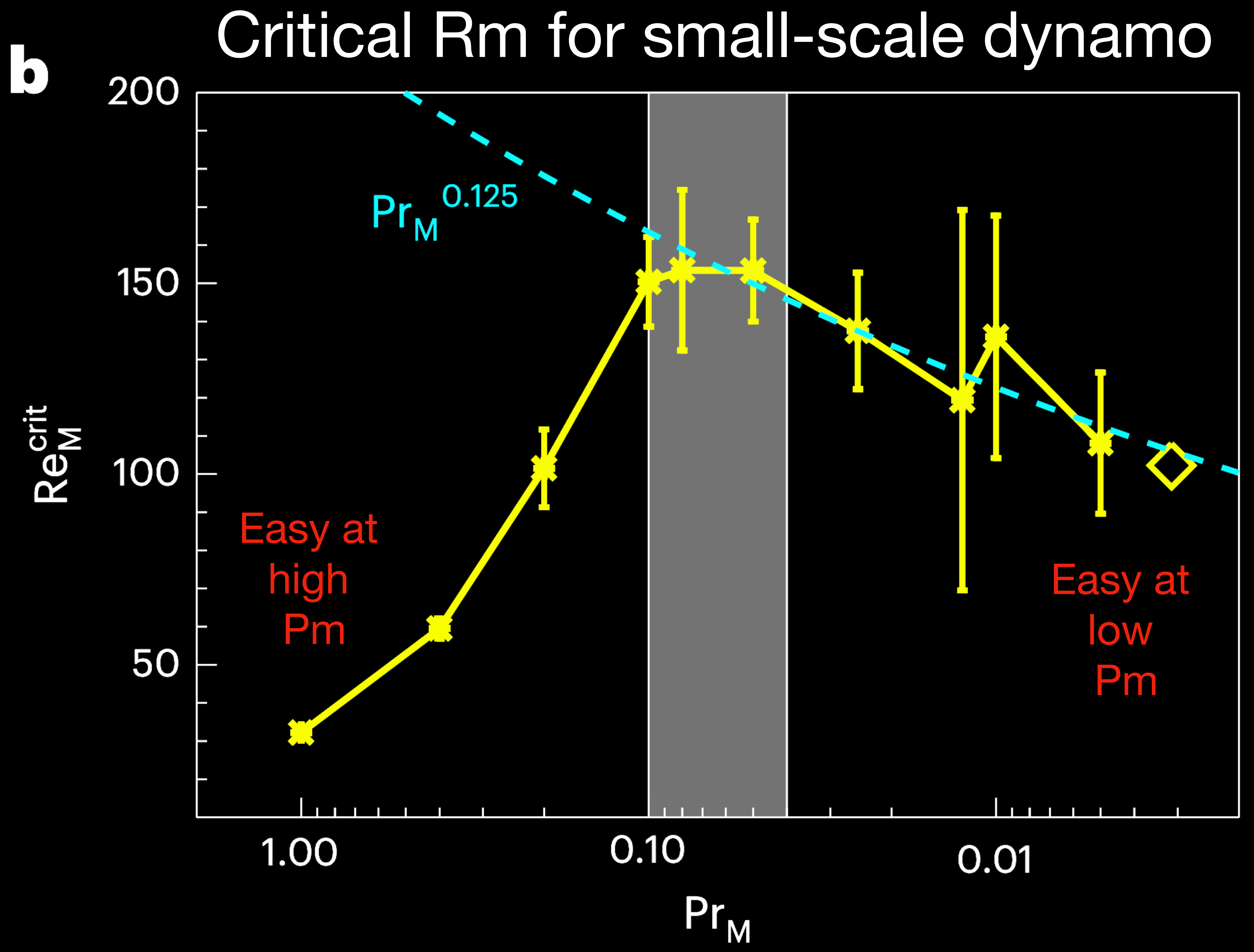
Ubiquitous across multiple phenomena / regimes

Extremely simple ingredients

1. $\nabla \otimes \mathbf{u}$

(dynamo is $B_j B_i \partial_i u_j$)

2. $$\mathrm{Rm} = \frac{U_0 L}{\eta} \gtrsim 100$$



Warnecke+2023. *Numerical evidence for a small-scale dynamo approaching solar magnetic Prandtl numbers.*

Simulations in this talk: 100s of no net-flux boxes

- 400 or so highly-modified version of finite volume code *FLASH*, second-order in space approximate Riemann solver with framework outlined in Bouchut+ (2010), tested in *FLASH* in Waagen+ (2011).

- Compressible non-helical, visco/resistive MHD turbulence driven with finite correlation time OU process on $L/2$.

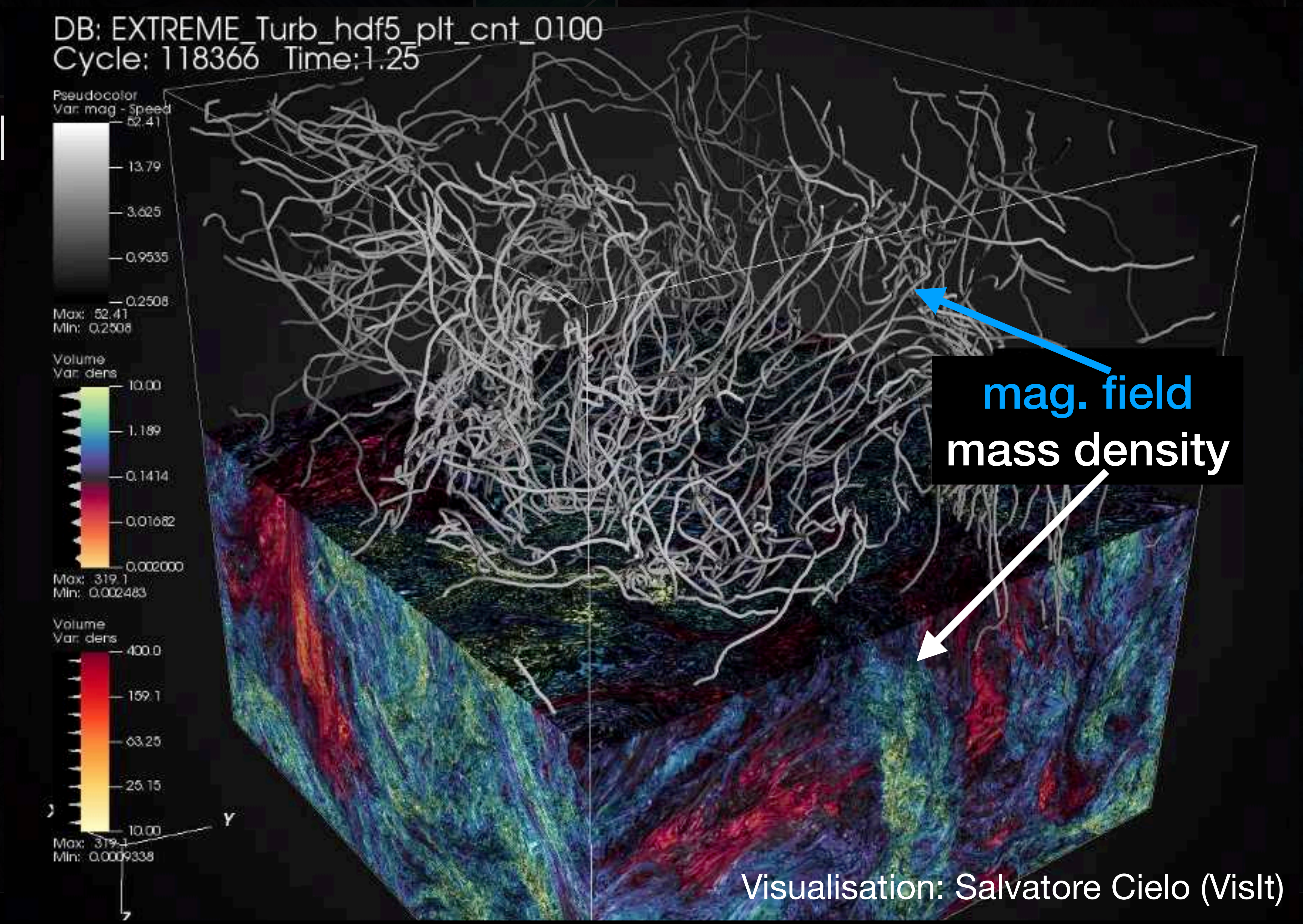
- No net magnetic flux. Pure turbulent magnetic fiel

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$d_t(\rho \mathbf{u}) + \nabla \cdot \mathbb{F} = \frac{1}{\mathrm{Re}} \nabla \cdot \sigma_{\mathrm{viscous}} + \rho \mathbf{f}$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{\mathrm{Rm}} \nabla^2 \mathbf{B}$$

$$\nabla \cdot \mathbf{B} = 0$$



Visualisation: Salvatore Cielo (VisIt)

Simulatio

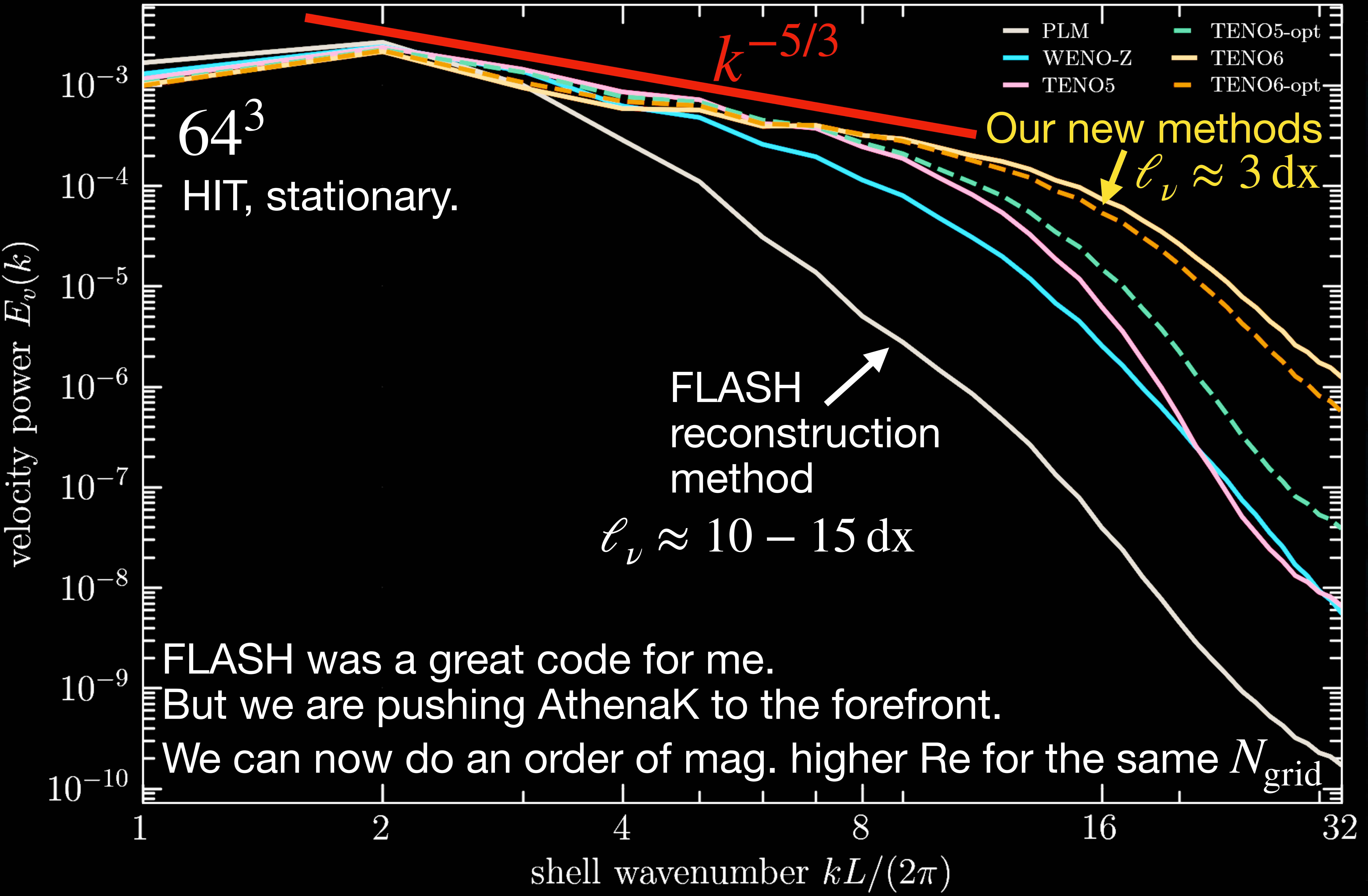
- 400 or so highl
  space approxin
  (2010), tested i
- Compressible r
  correlation time
- No net magneti

$$\partial_t \rho + \nabla \cdot ($$

$$d_t(\rho \mathbf{u}) + \nabla$$

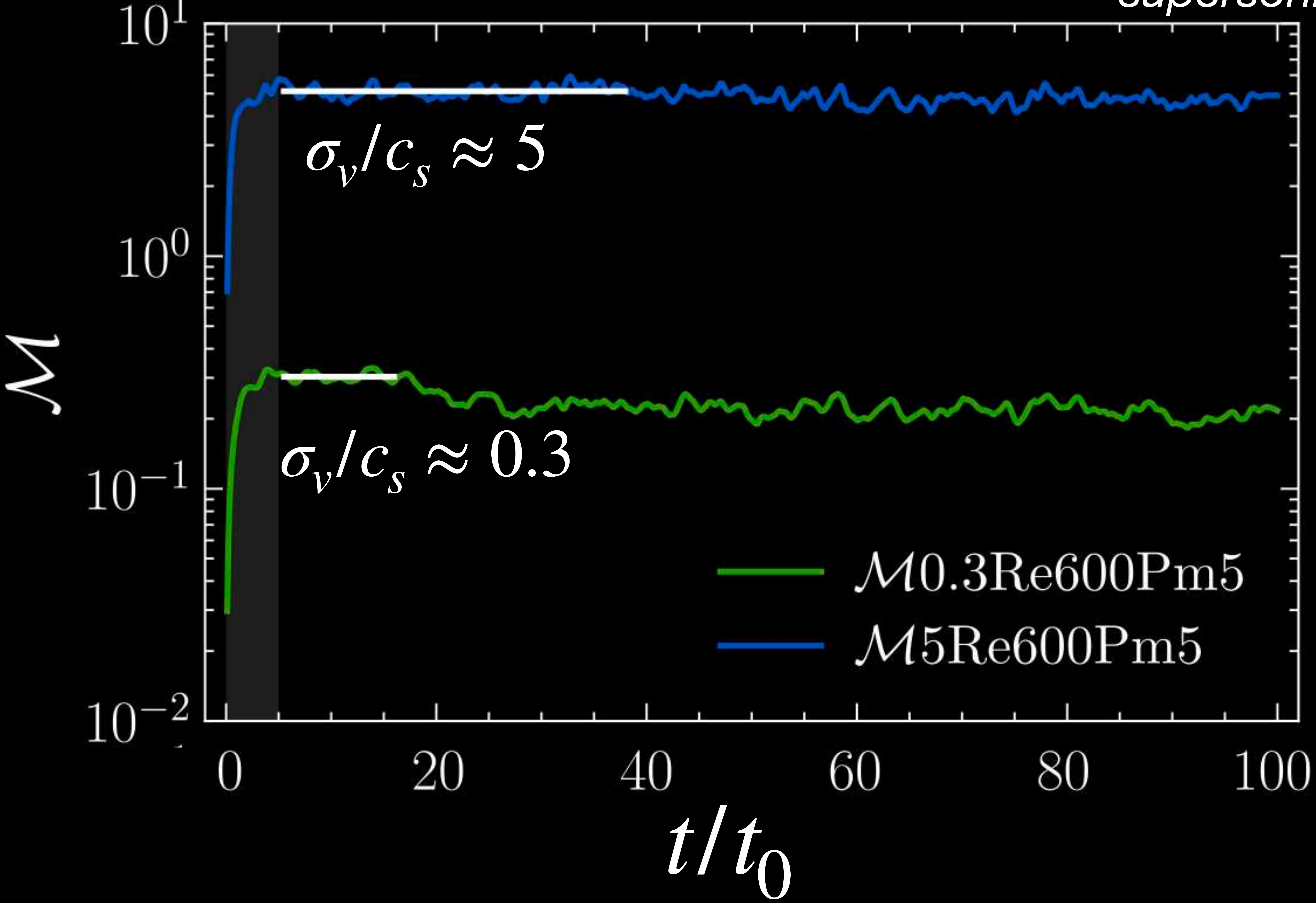
$$\partial_t \mathbf{B} = \nabla \times$$

$$\nabla \cdot \mathbf{B} = 0$$



Simulations in this talk

Kriel, **Beattie**+ (2025). *Fundamental scales II: the kinematic stage of the supersonic dynamo*



$$\mathrm{Pm} = \frac{\nu}{\eta} \propto \frac{T^4}{n_e}$$

$$\mathrm{Rm} = \frac{U_0 L}{\eta}$$

$$\mathrm{Re} = \frac{U_0 L}{\nu}$$

$$\mathscr{M} = \frac{\langle u^2 \rangle^{1/2}}{\langle c_s \rangle}$$

Simulations in this talk

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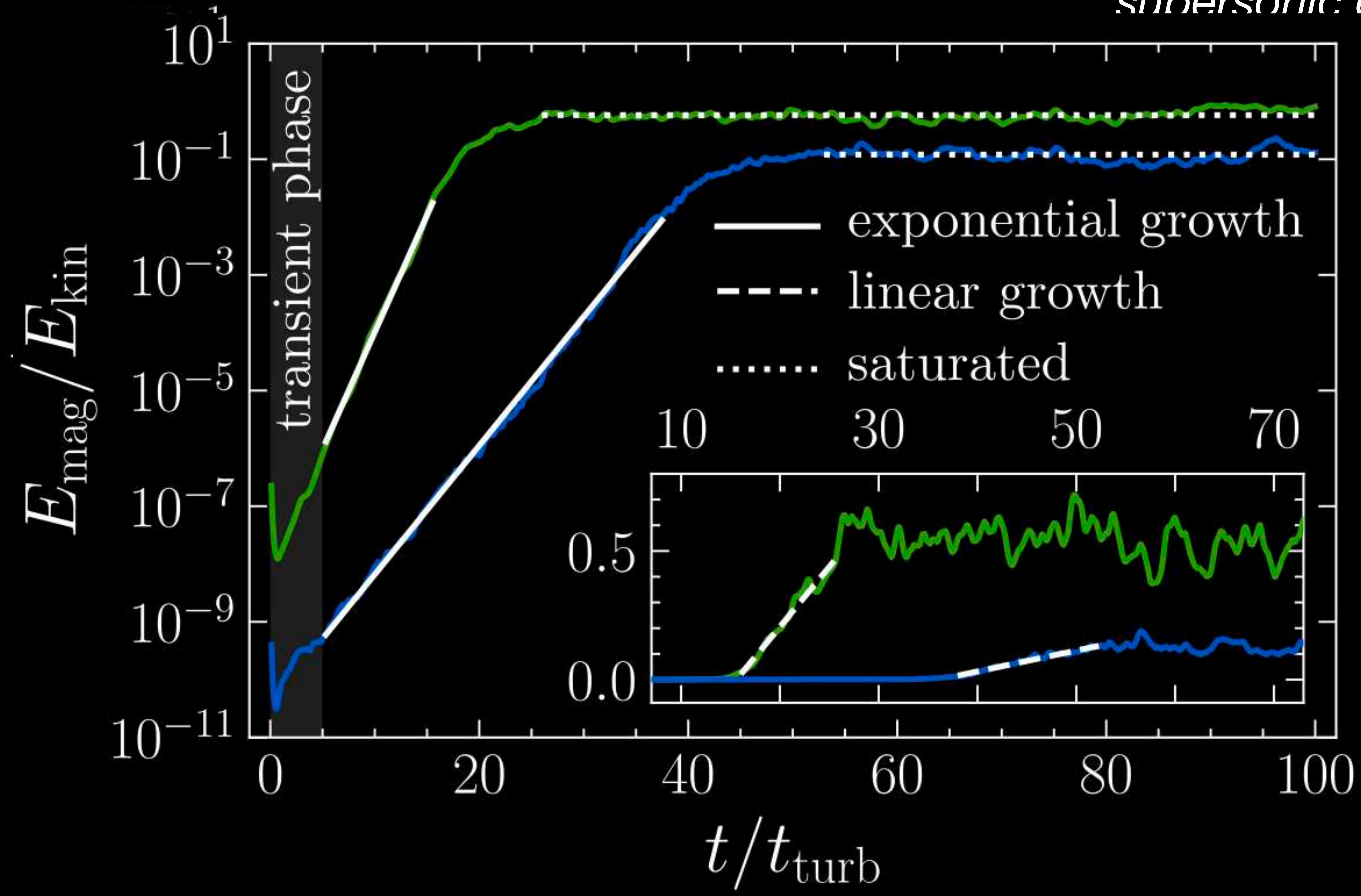

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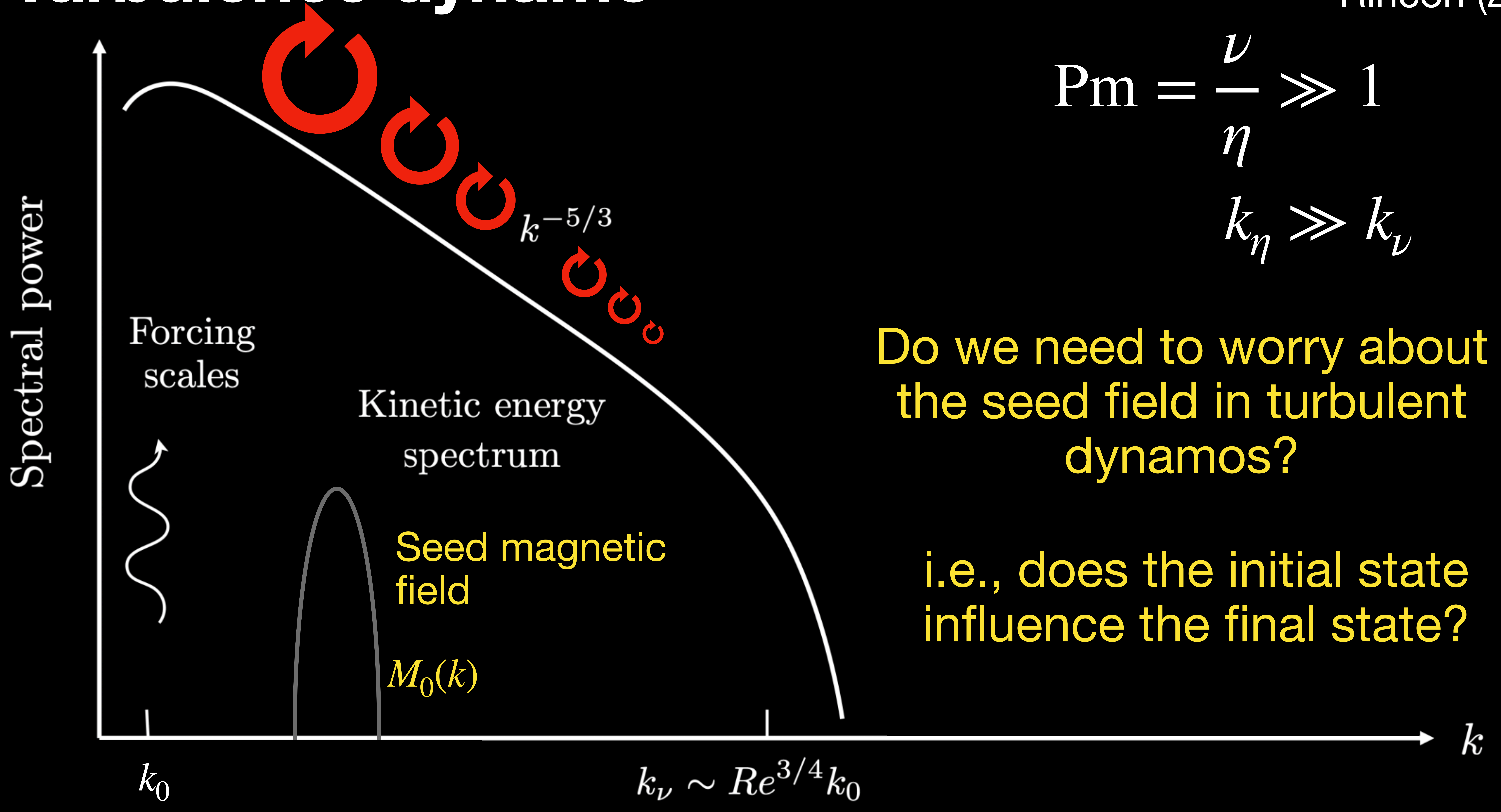
Turbulence dynamo

Modified from Rincon (2019)



$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

Do we need to worry about the seed field in turbulent dynamos?

i.e., does the initial state influence the final state?

Turbulence dynamo

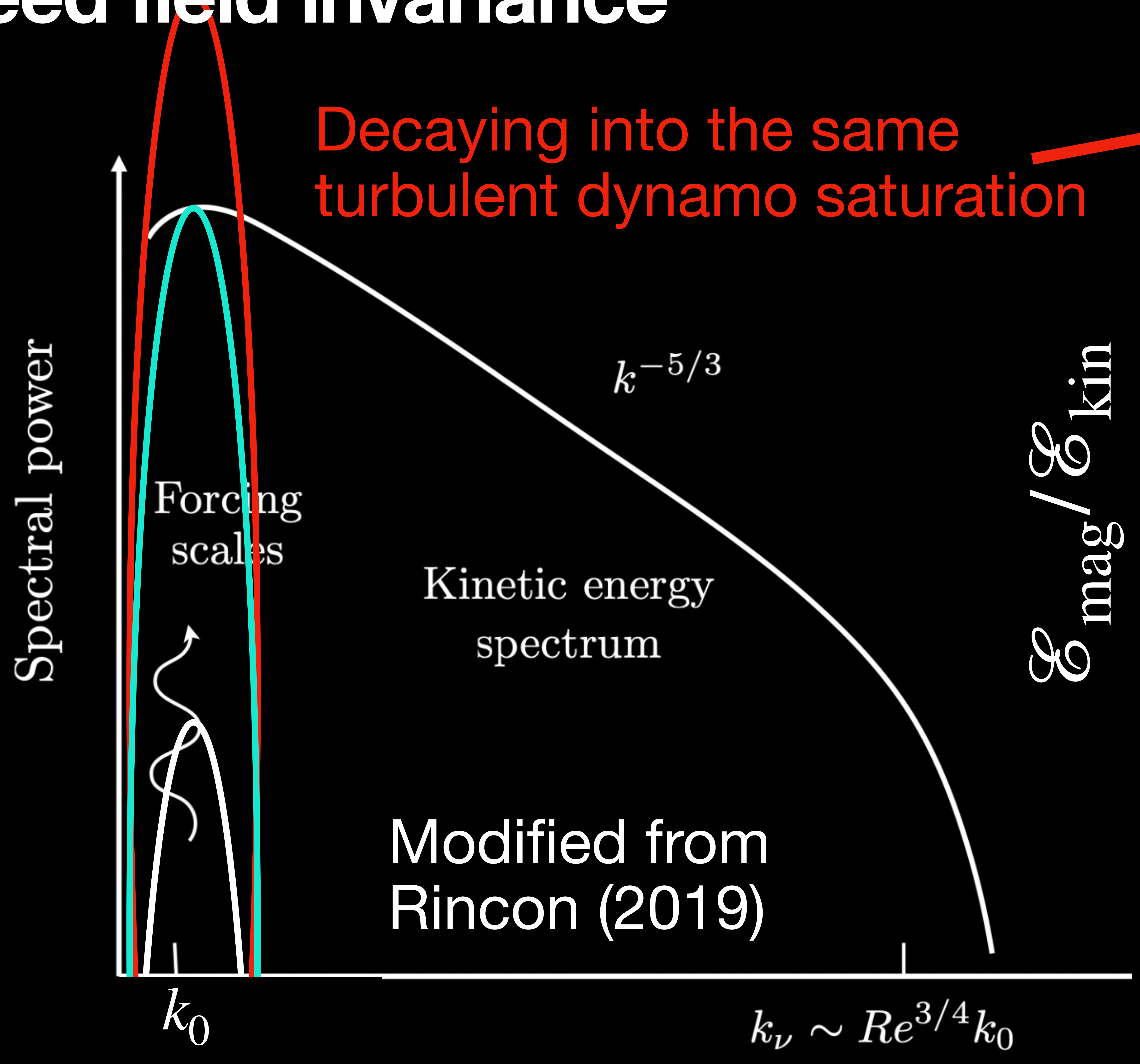
Seed field invariance

Modified from Rincon (2019)

Turbulence dynamo

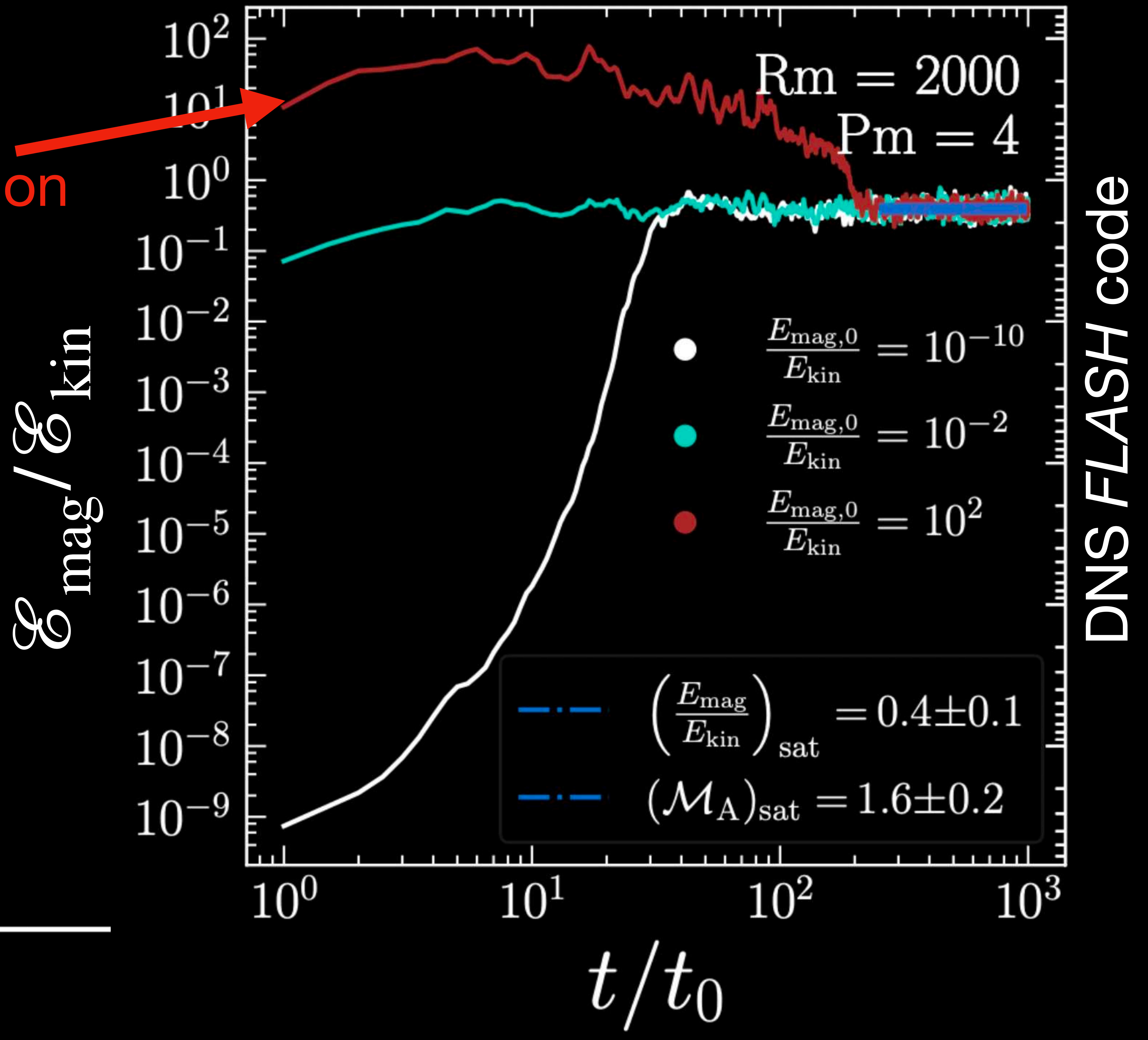
Seed field invariance

Decaying into the same turbulent dynamo saturation



Modified from Rincon (2019)

Invariant to amplitude



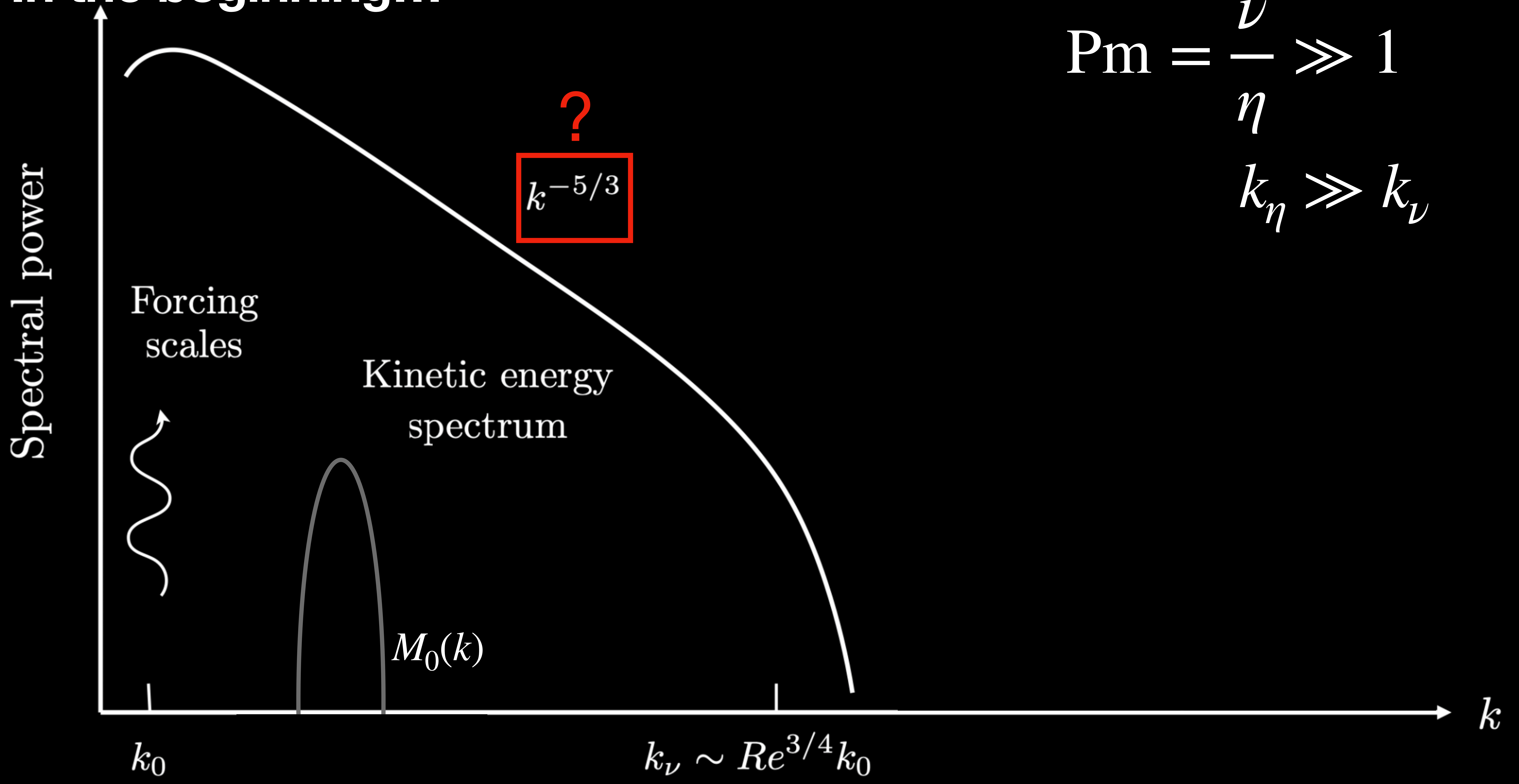
Beattie+ (2023). *Growth or Decay I: Universality of the turbulent dynamo saturation*

Turbulence dynamo
In the beginning…

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$



Turbulence dynamo
Kinematic regime: the sub-viscous range

Derived from $k^{-5/3}$ velocity spectrum

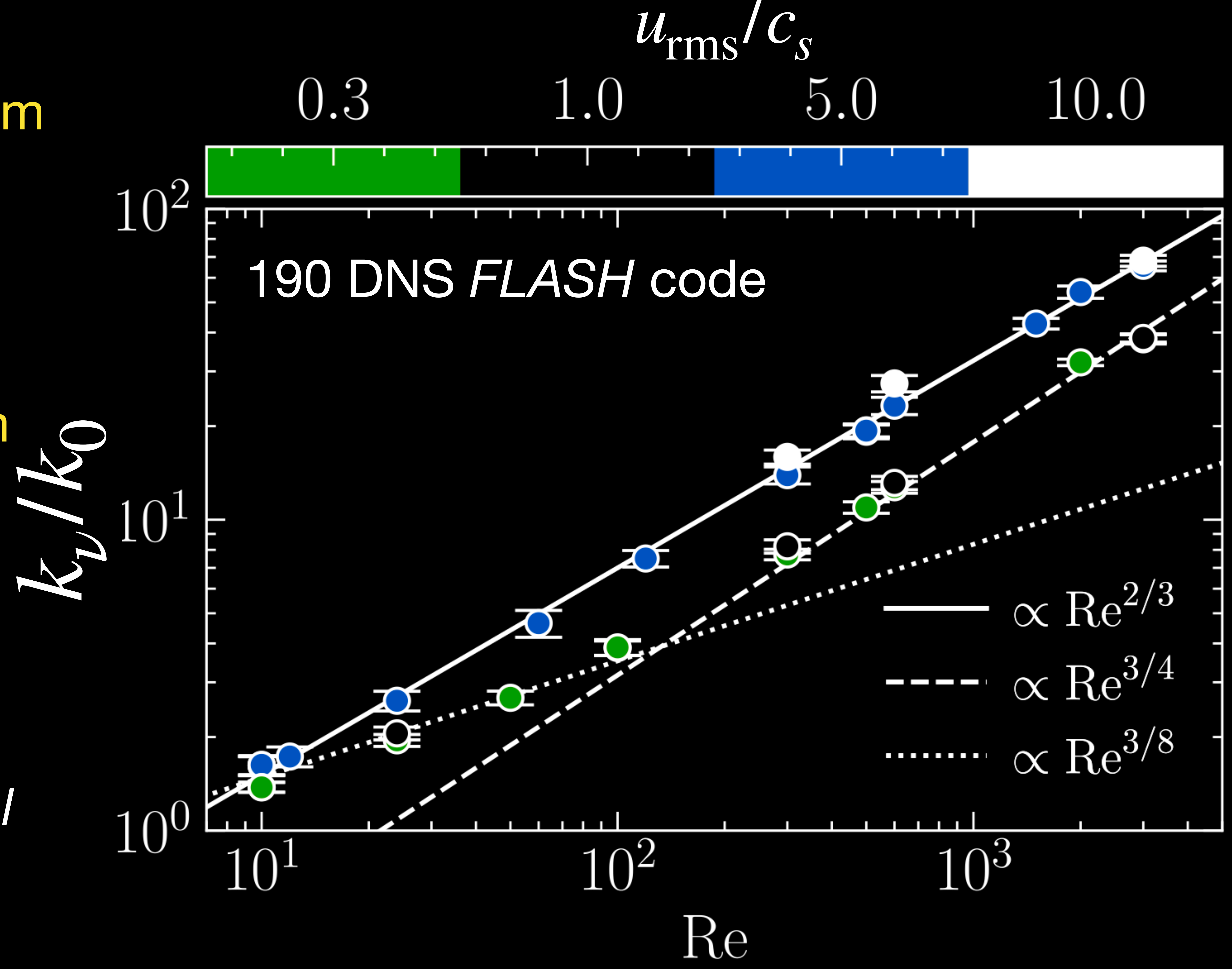
$\mathcal{M} \ll 1$ $\quad k_\nu \sim \mathrm{Re}^{3/4}$

Kolmogorov41

Derived from k^{-2} velocity spectrum

$\mathcal{M} \gg 1$ $\quad k_\nu \sim \mathrm{Re}^{2/3}$

Schober+(2015)
Beattie+(2025)



Kriel, **Beattie**+ (2025). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

Turbulence dynamo

Kinematic regime: the sub-viscous range

Kriel, **Beattie**+ (2025). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

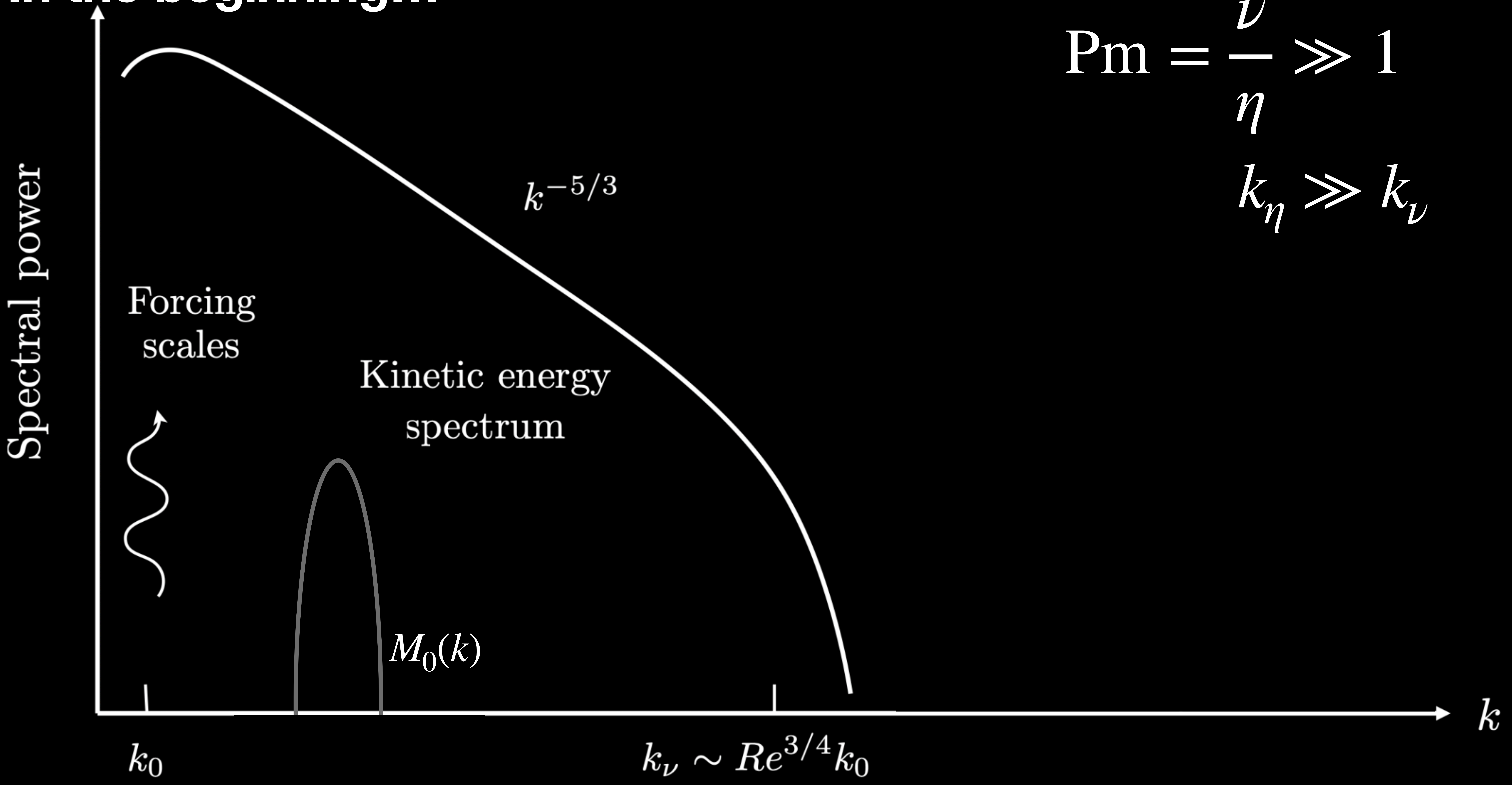
Turbulence dynamo

In the beginning…

Modified from Rincon (2019)

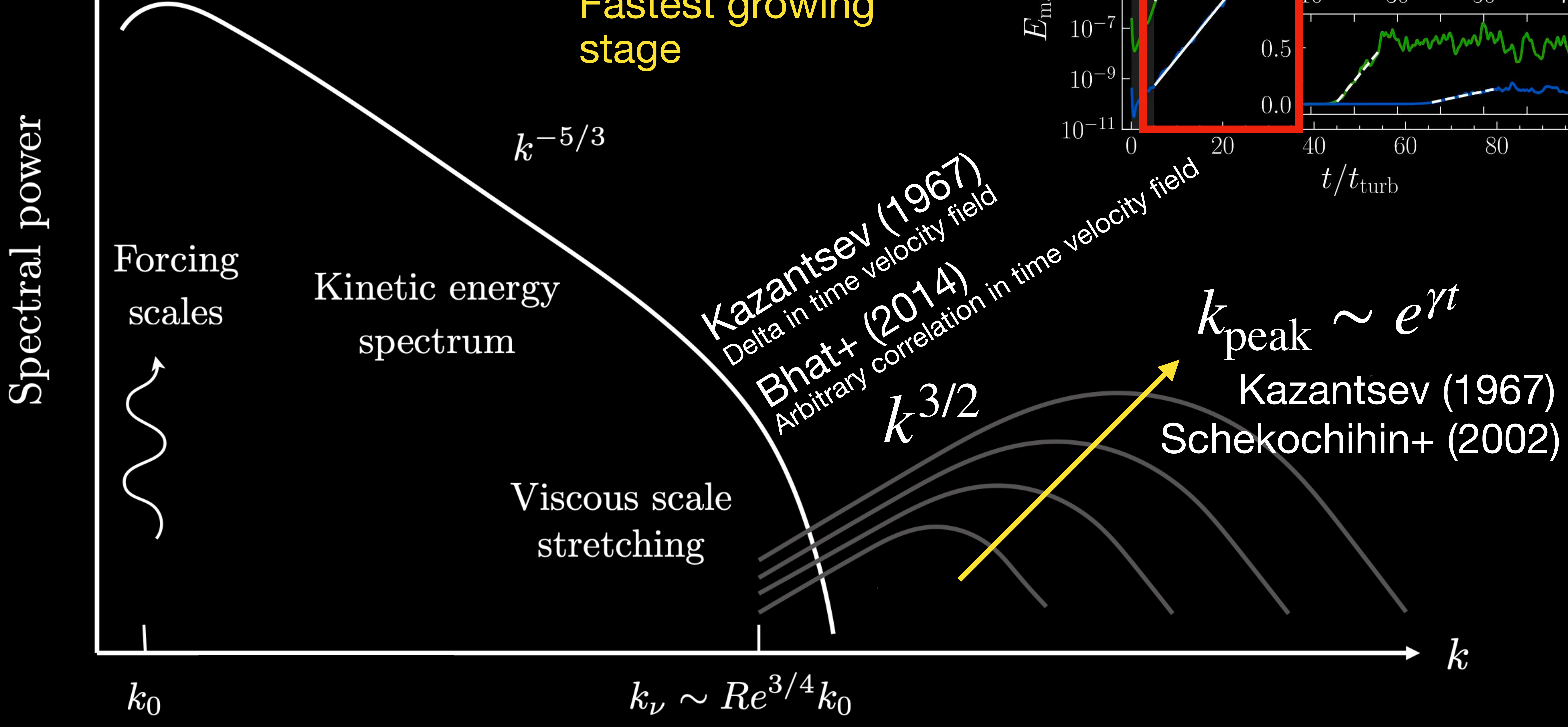
$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

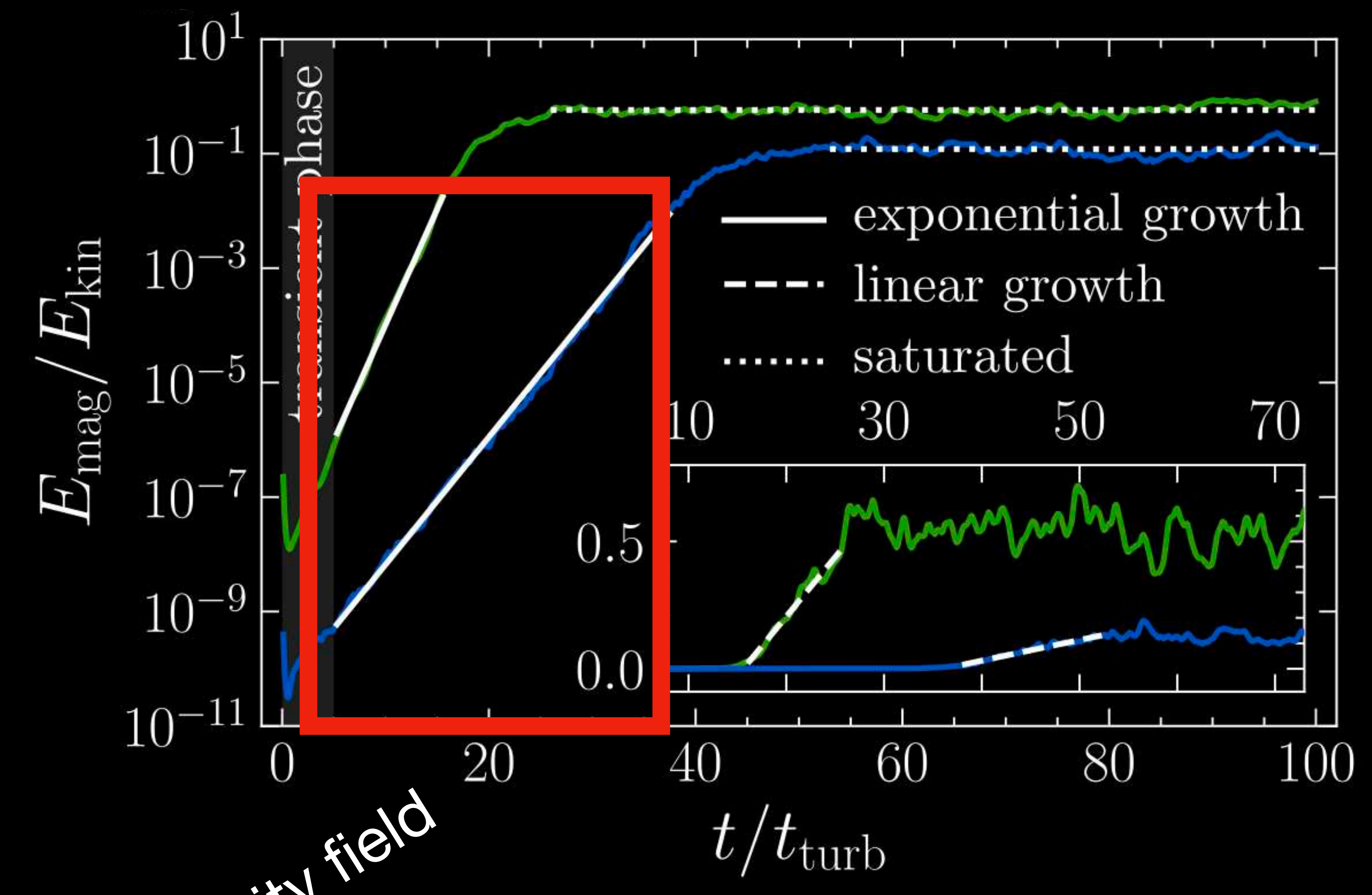
$$k_\eta \gg k_\nu$$



Turbulence dynamo

First growth stage: diffusion-free kinematic regime





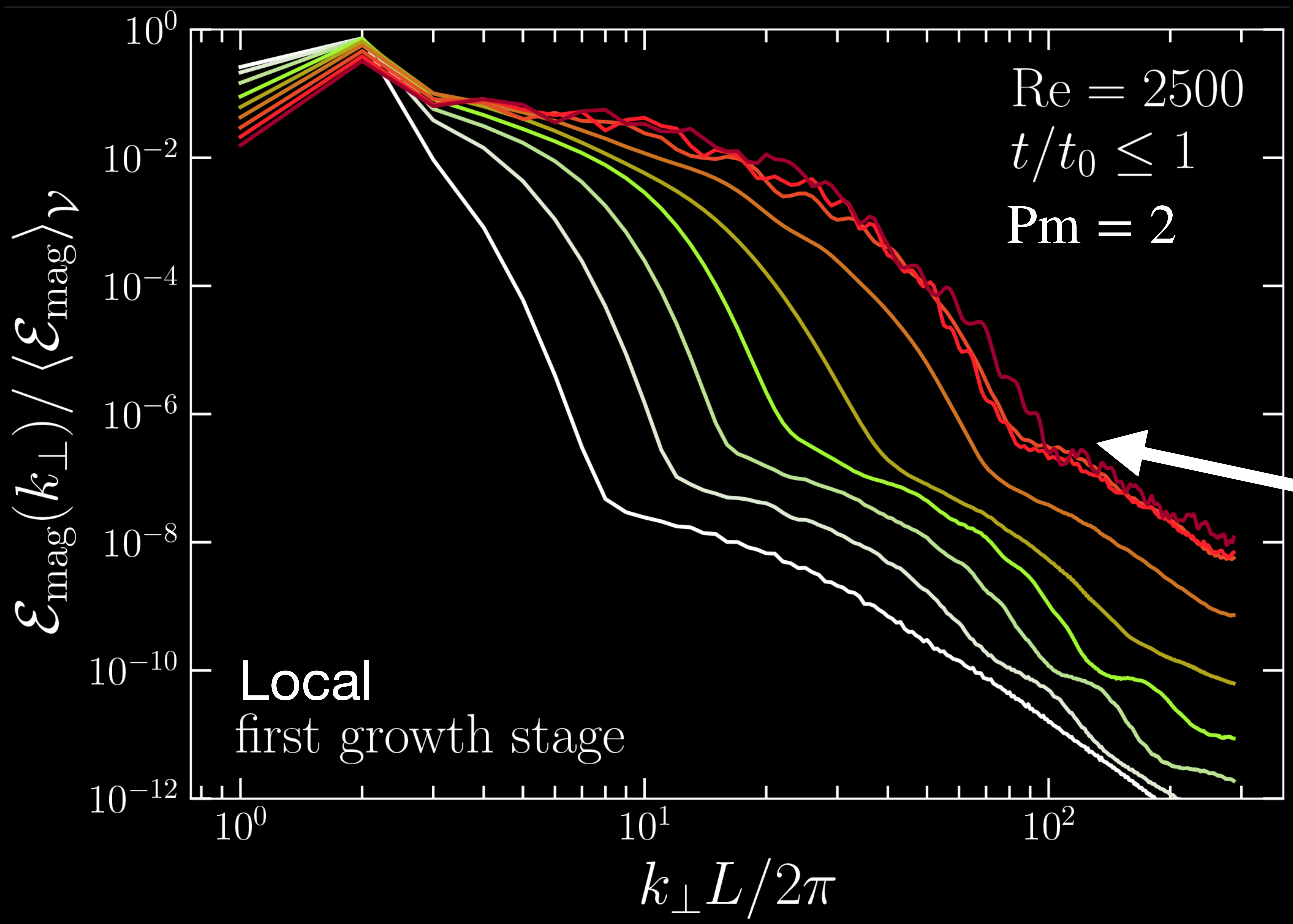
Turbulence dynamo

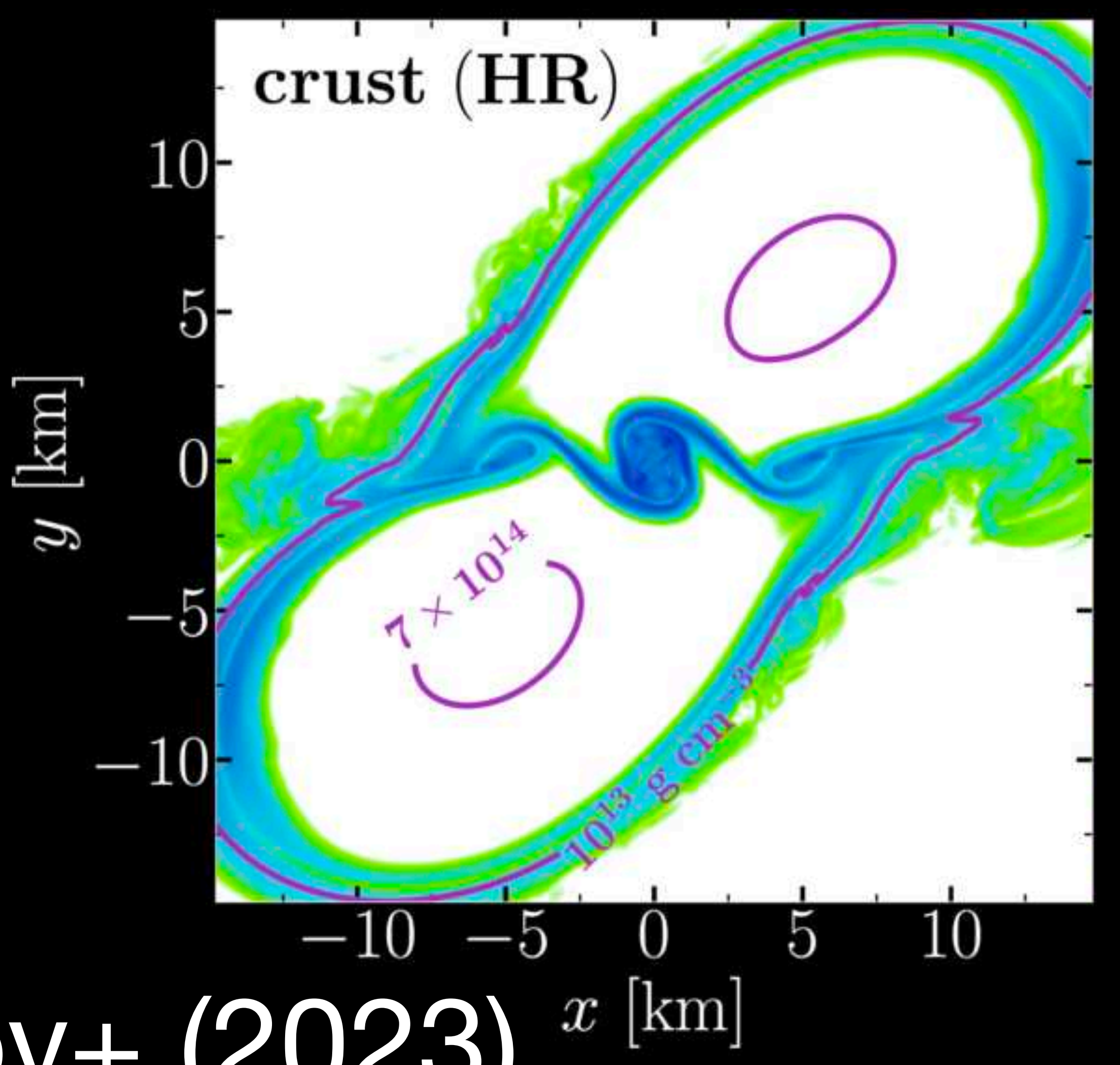
First growth stage: diffusion-free kinematic regime

Modified from Rincon (2019)

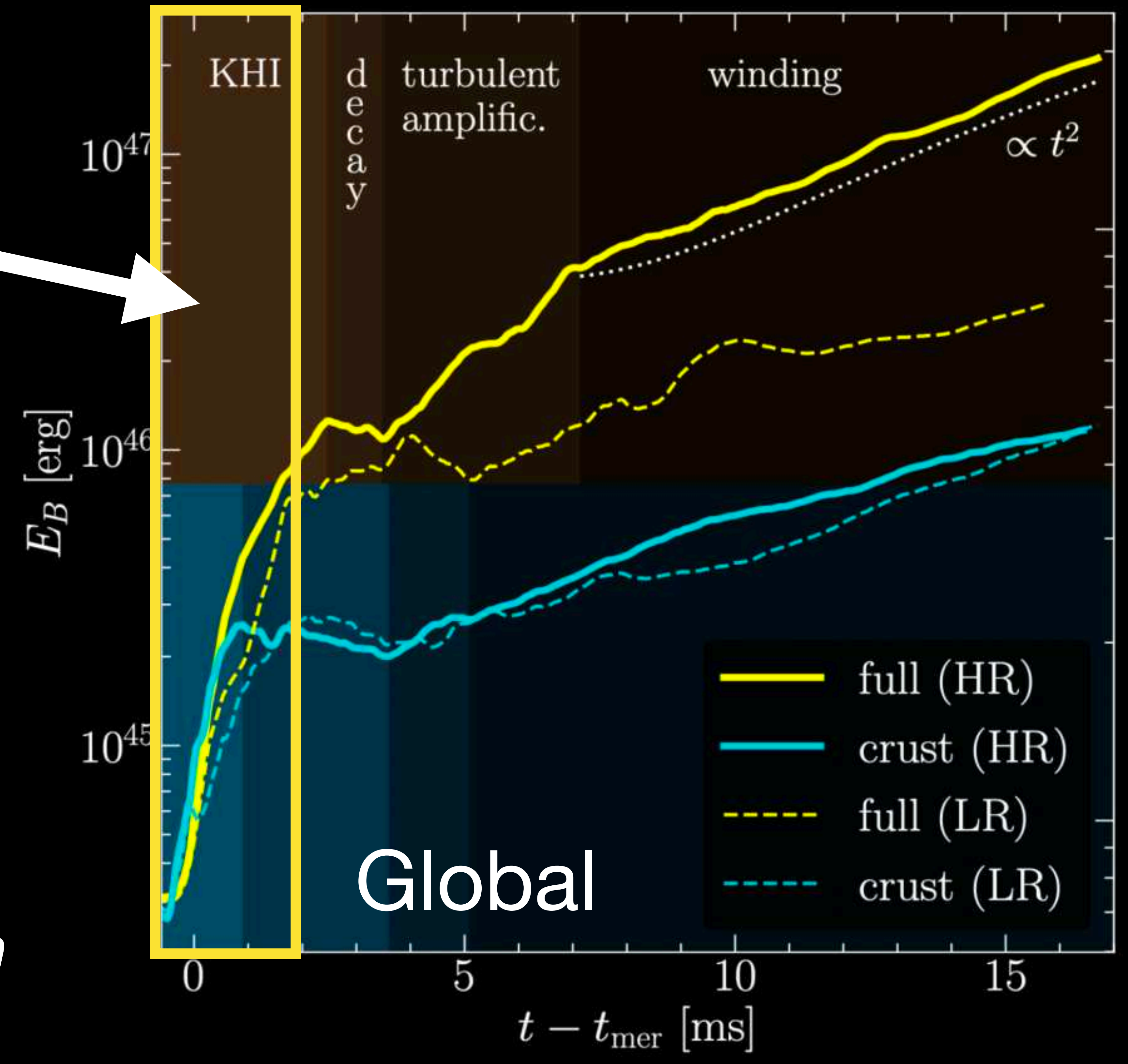
Turbulence dynamo

Local KHI dynamo simulations





Chabanov+ (2023)



<u>Beattie</u>+ (in prep.) The Kinematic Kelvin-Helmholtz Dynamo Between Merging Compact Objects.

Turbulence dynamo
The engine of the kinematic dynamo

Growth rate dominated by the smallest possible scales of the flow gradients

$$\gamma = \langle \hat{B}_i \hat{B}_j \partial_j u_i \rangle \sim u_\nu / \ell_\nu \sim 1/t_\nu,$$

$$u_\ell / \ell \sim \varepsilon^{1/3} \ell^{-2/3}, \quad t_\ell \sim \ell^{2/3}.$$

put in units of outer scale turnover time $t_0 = \ell_0 / u_0$

$$t_0 \gamma \sim t_0 / t_\nu,$$

and to summarise,

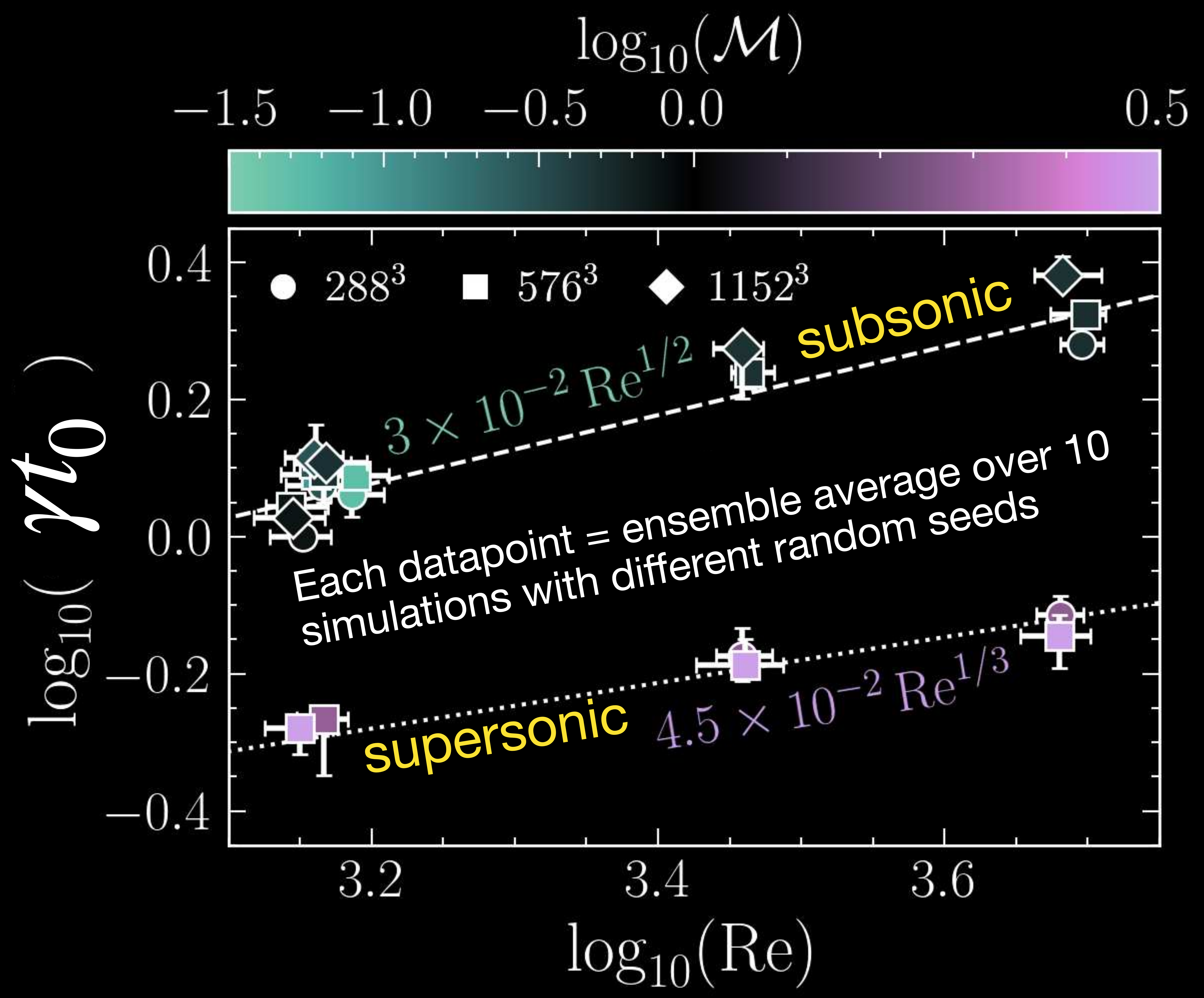
K41 spectrum (incompresible)

$$t_0 \gamma \sim (\ell_0 / \ell_\nu)^{2/3} \sim \left(\mathrm{Re}^{3/4}\right)^{2/3} \sim \mathrm{Re}^{1/2},$$

Burgers spectrum (supersonic)

$$t_0 \gamma \sim (\ell_0 / \ell_\nu)^{1/2} \sim \left(\mathrm{Re}^{2/3}\right)^{1/2} \sim \mathrm{Re}^{1/3}.$$

Turbulence dynamo
Confronting γ with data



The engine of the kinematic dynamo is k_ν

K41 spectrum (incompressible)

$$t_0\gamma \sim \mathrm{Re}^{1/2},$$ ✓

Burgers spectrum (supersonic)

$$t_0\gamma \sim \mathrm{Re}^{1/3}.$$ ✓

Turbulence dynamo

First growth stage: diffusion-free kinematic regime

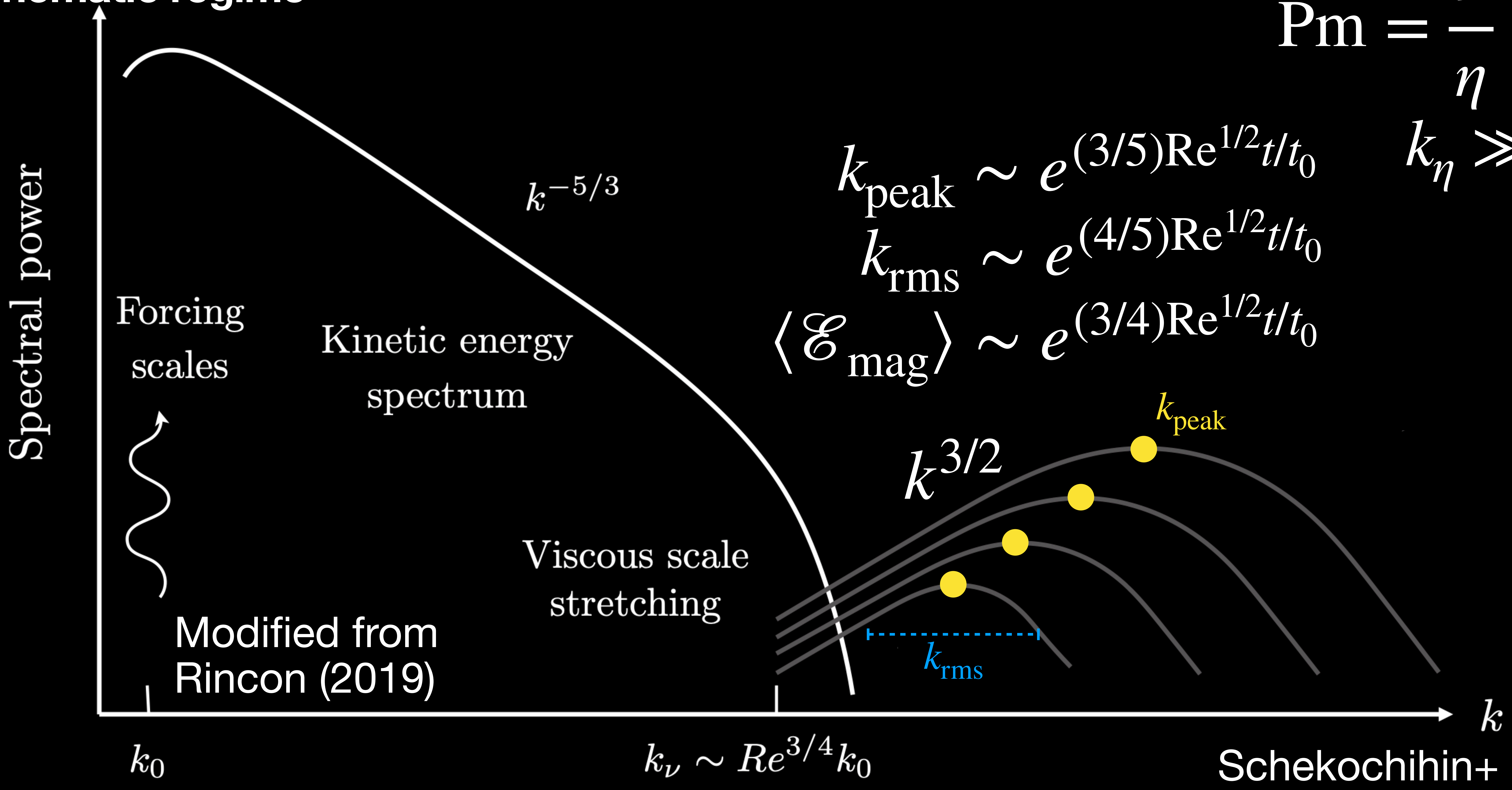
$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

$$k_{\mathrm{peak}} \sim e^{(3/5)\mathrm{Re}^{1/2} t/t_0}$$

$$k_{\mathrm{rms}} \sim e^{(4/5)\mathrm{Re}^{1/2} t/t_0}$$

$$\langle \mathcal{E}_{\mathrm{mag}} \rangle \sim e^{(3/4)\mathrm{Re}^{1/2} t/t_0}$$

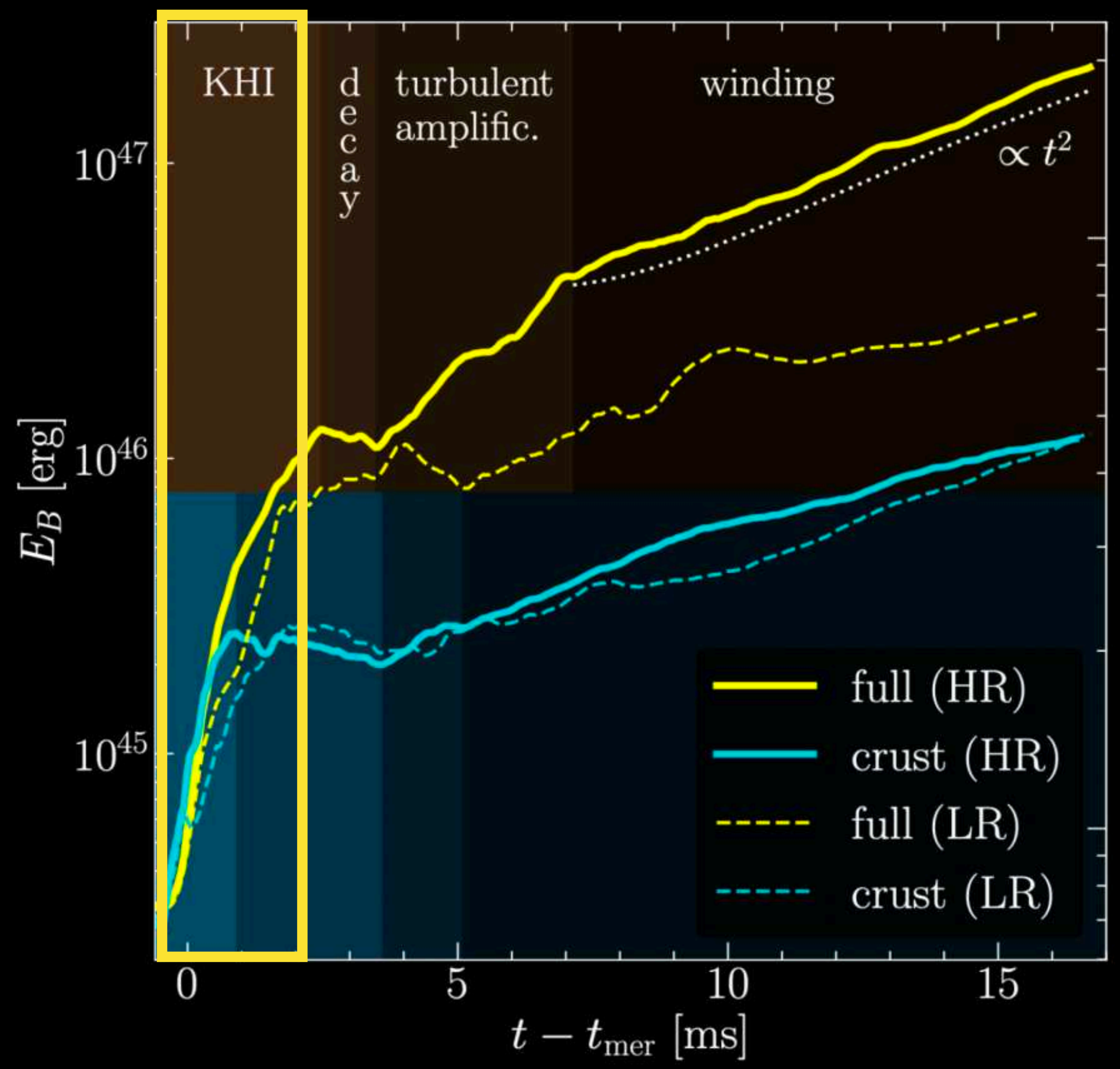


Schekochihin+ (2002)

Turbulence dynamo

Local KHI dynamo simulations

Chabanov+ (2023)



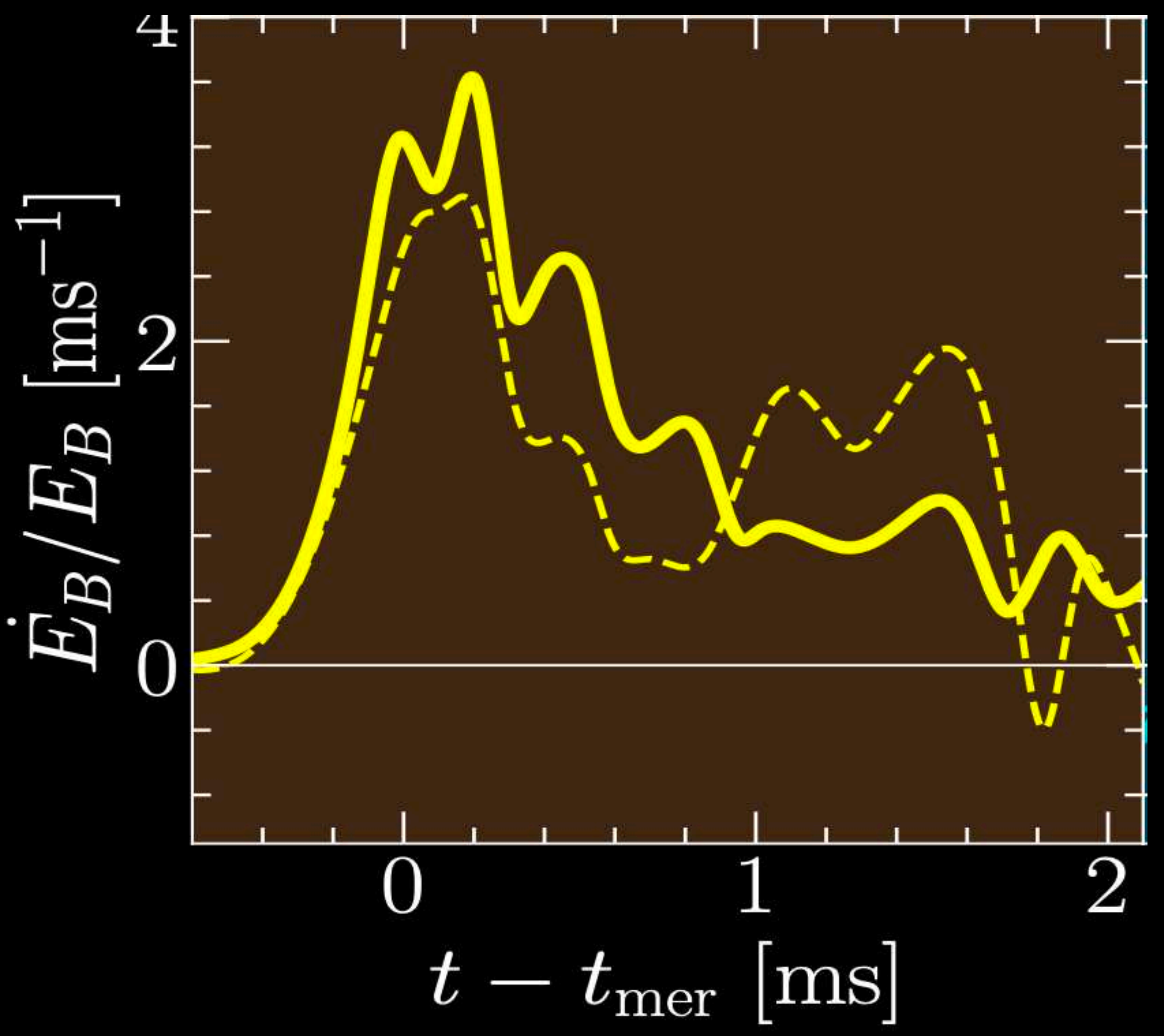
Neutrinos create a bulk viscosity

$$\mathrm{Re} \sim 10^{6+}$$

$$\Updownarrow$$

$$\gamma \sim 10^{3}\,(\mathrm{ms})^{-1}$$

An extreme challenge for global sims



(bulk viscosity can act as an effective shear viscosity for the turbulent dynamo)

***Beattie**+ (2025, MNRAS) Taking control of compressible modes: bulk viscosity and the turbulent dynamo*

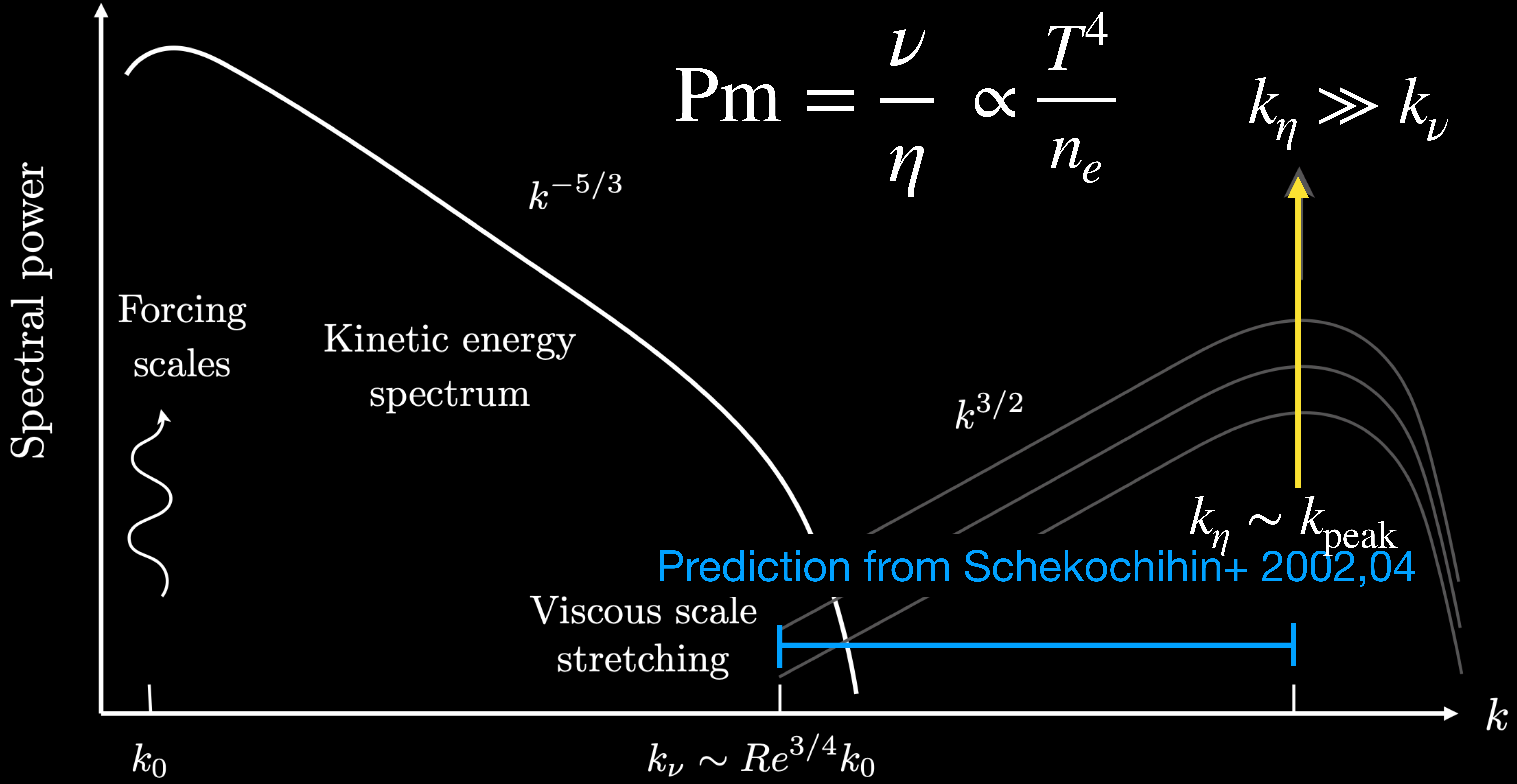
Turbulence dynamo

First growth stage: kinematic regime & the sub-viscous range

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \propto \frac{T^4}{n_e}$$

$$k_\eta \gg k_\nu$$



Turbulence dynamo

Kinematic regime: the sub-viscous range

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

stretching at the viscous scale

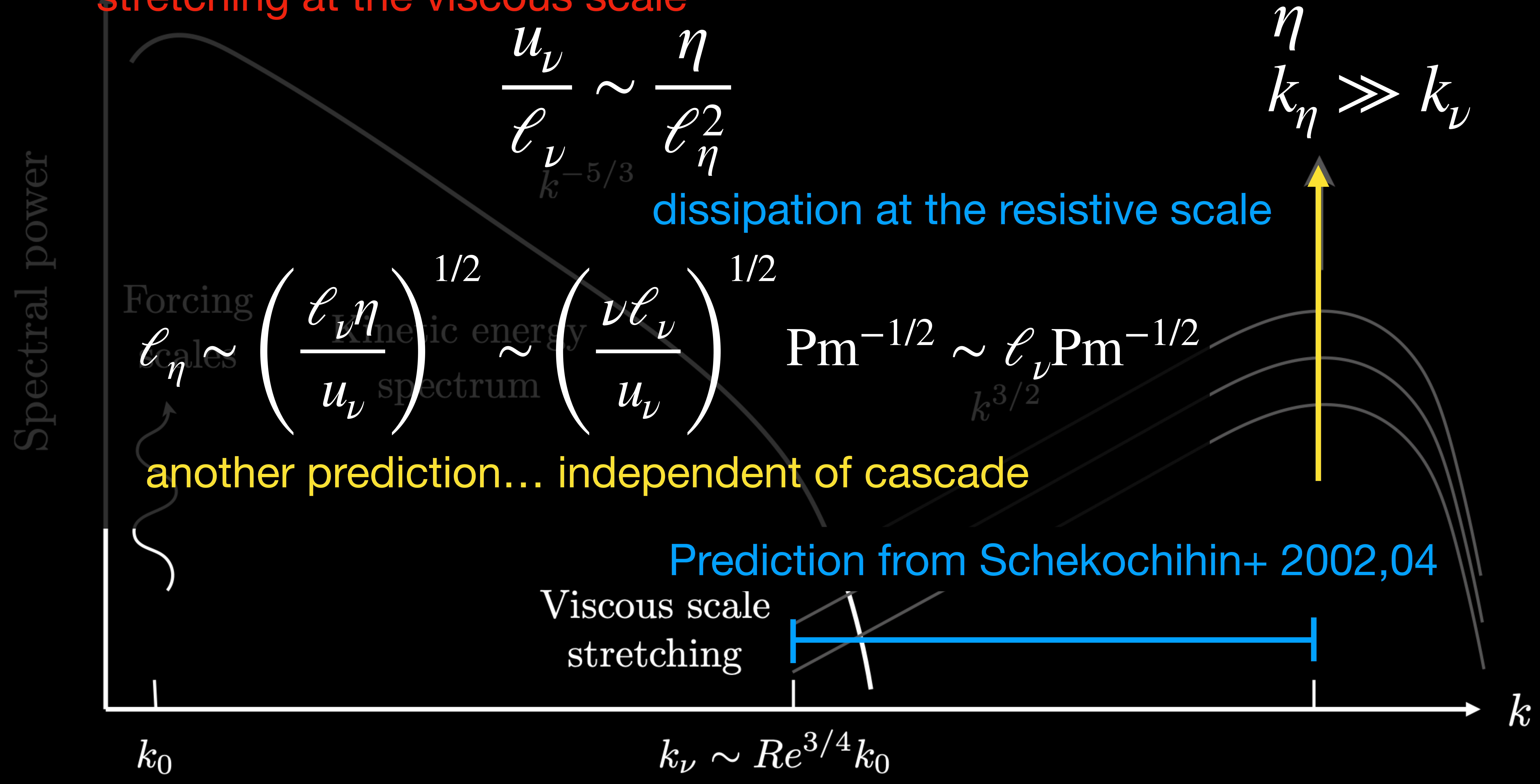
$$\frac{u_\nu}{\ell_\nu} \sim \frac{\eta}{\ell_\eta^2}$$

dissipation at the resistive scale

$$\ell_\eta \sim \left(\frac{\ell_\nu \eta}{u_\nu}\right)^{1/2} \sim \left(\frac{\nu \ell_\nu}{u_\nu}\right)^{1/2} \mathrm{Pm}^{-1/2} \sim \ell_\nu \mathrm{Pm}^{-1/2}$$

another prediction… independent of cascade

Prediction from Schekochihin+ 2002,04



Turbulence dynamo

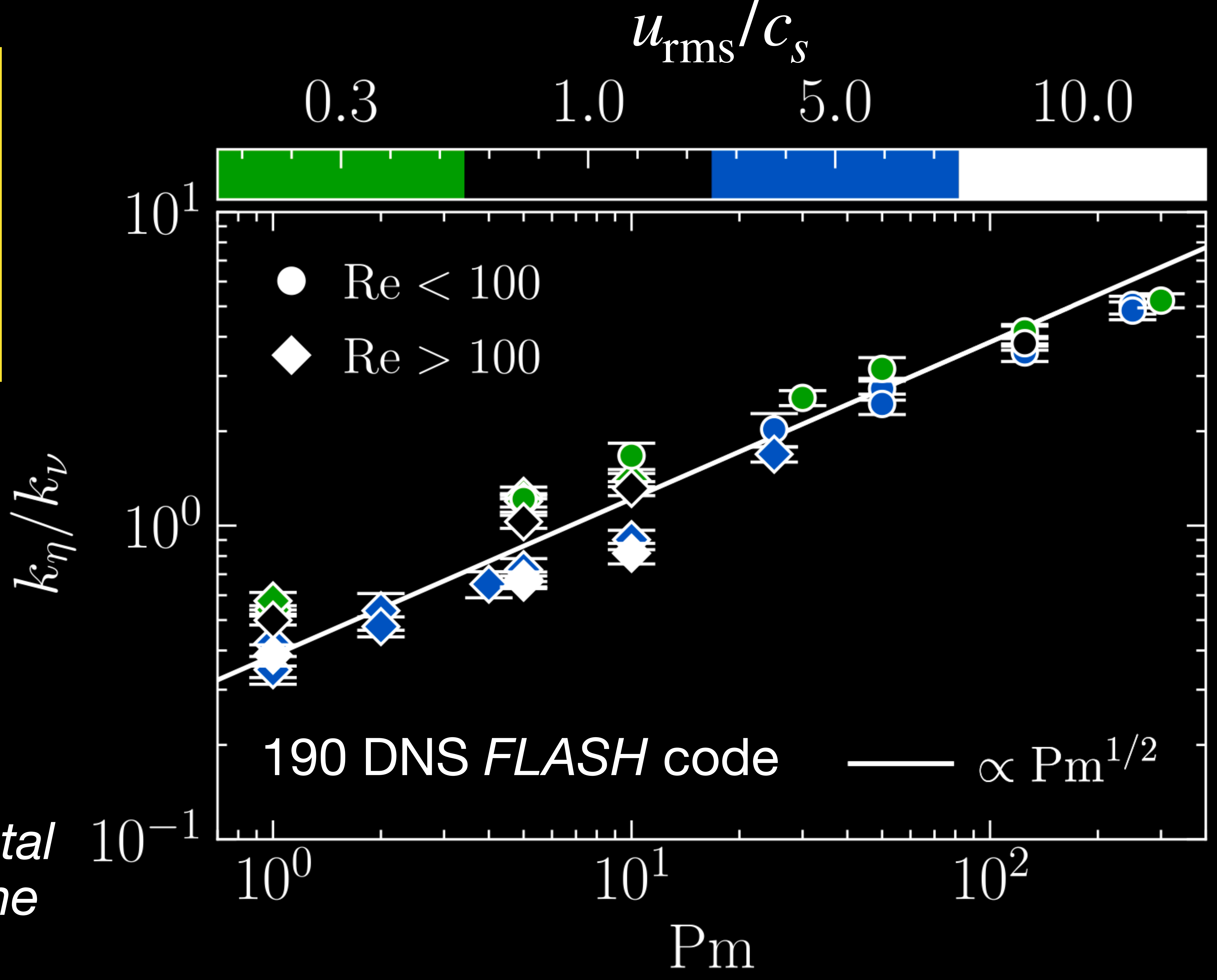
Most of the energy is in the sub-viscous modes!

stretching at the viscous scale

$$\frac{u_\nu}{\ell_\nu} \sim \frac{\eta}{\ell_\eta^2}$$

dissipation at the resistive scale

$$\frac{k_\eta}{k_\nu} \sim \mathrm{Pm}^{1/2}$$



Kriel, **Beattie**+ (2025). *Fundamental scales II: the kinematic stage of the supersonic dynamo*

How fast do the sub-viscous modes populate?

(Assuming $M_0(k) = M_0\delta(k - k_0)$)

Cold ISM

CNM, $\mathrm{Re} \sim 10^{10}$, $\mathrm{Pm} \sim 10^4$, $t_0 \sim 10$ pc/ 3 km s^{-1} $\sim$ Myr

$\Delta t \ll t_0$

$$\Delta t \sim \ln(\mathrm{Re}^{3/4}\mathrm{Pm}^{1/2})\gamma^{-1} \implies \Delta t \sim 20\gamma^{-1} \implies \Delta t \sim 200 \text{ years},$$

~ 8 orders of mag.

Hot ISM and ICM

HIM, $\mathrm{Re} \sim 10^{2}$, $\mathrm{Pm} \sim 10^{20}$, $t_0 \sim 30$ pc/ 10 km s^{-1} $\sim$ Myr

$$\Delta t \sim 25\gamma^{-1} \implies \Delta t \sim \text{Myr}, \Delta t \sim t_0$$

~ 11 orders of mag.

ICM, $\mathrm{Re} \sim 10^{2}$, $\mathrm{Pm} \sim 10^{29}$, $t_0 \sim 100$ kpc/ 200 km s^{-1} $\sim 10^2$ Myr

$$\Delta t \sim 40\gamma^{-1} \implies \Delta t \sim 400 \text{ Myr}, \Delta t \sim t_0$$

~ 17 orders of mag.

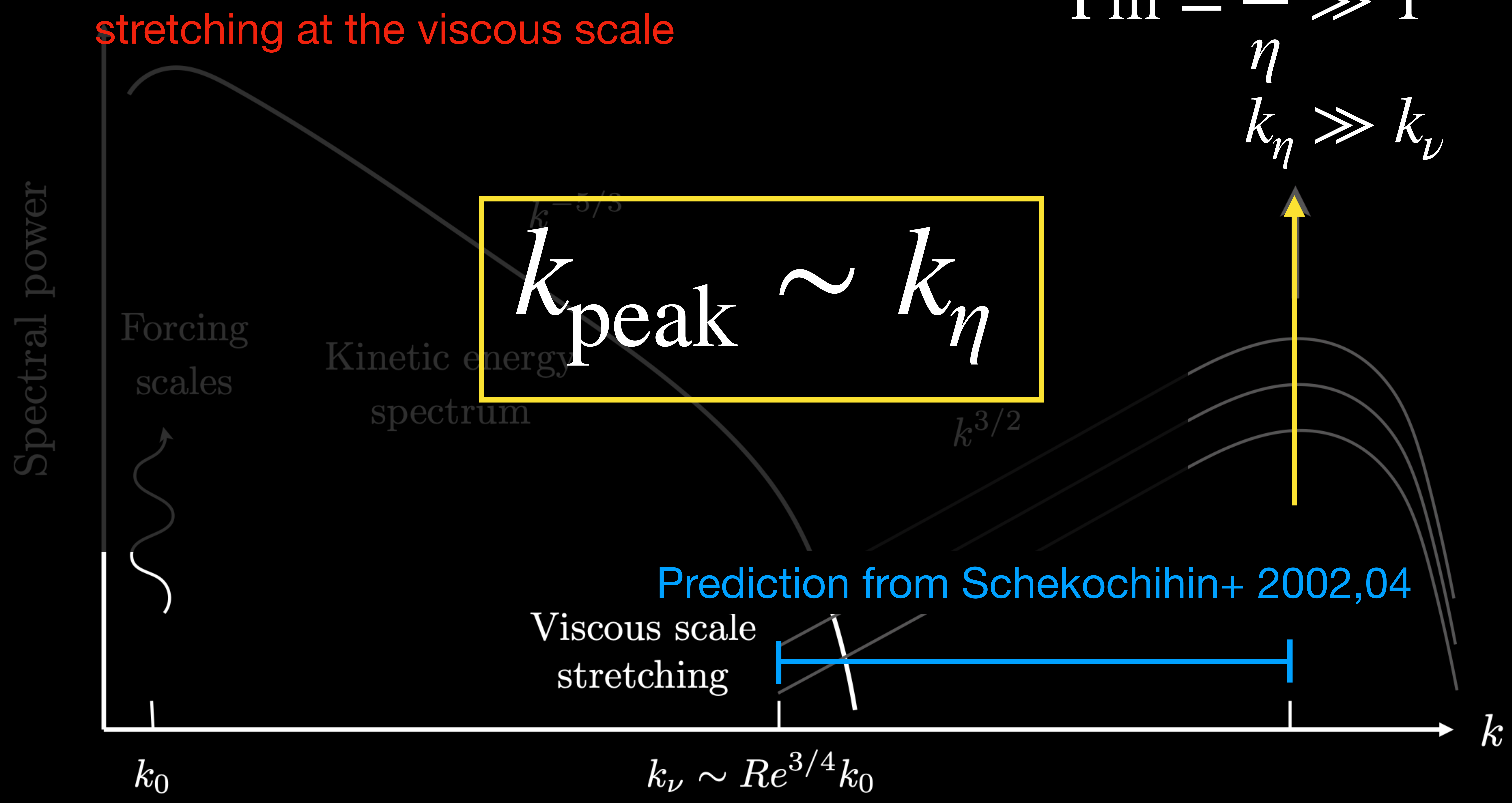
Turbulence dynamo

Kinematic regime: the sub-viscous range

Modified from Rincon (2019)

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

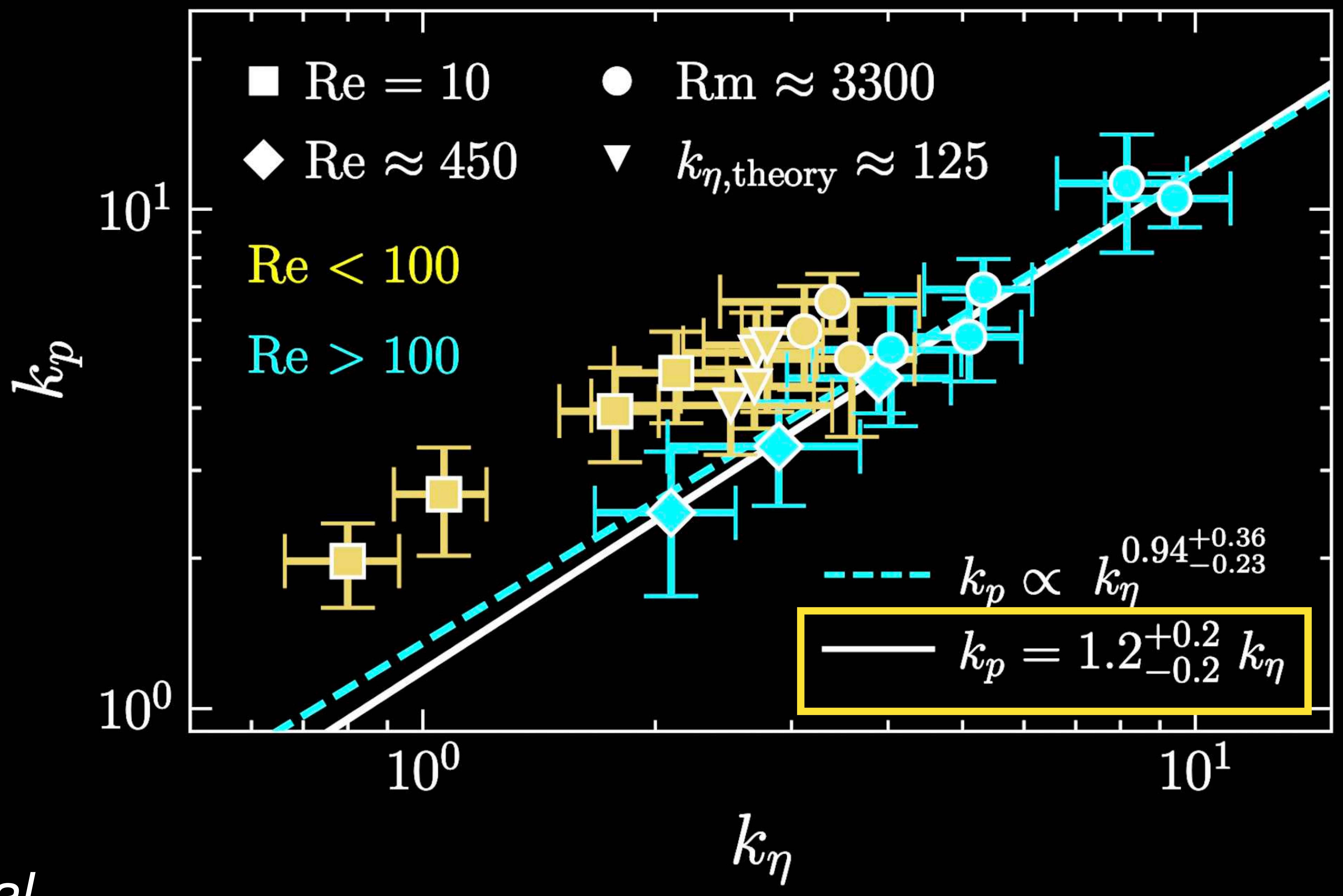
$$k_\eta \gg k_\nu$$



Turbulence dynamo
Most of the energy on the resistive scale!

The magnetic energy is concentrated at the smallest scales!

$$k_{\text{peak}} \sim k_\eta$$ ✓

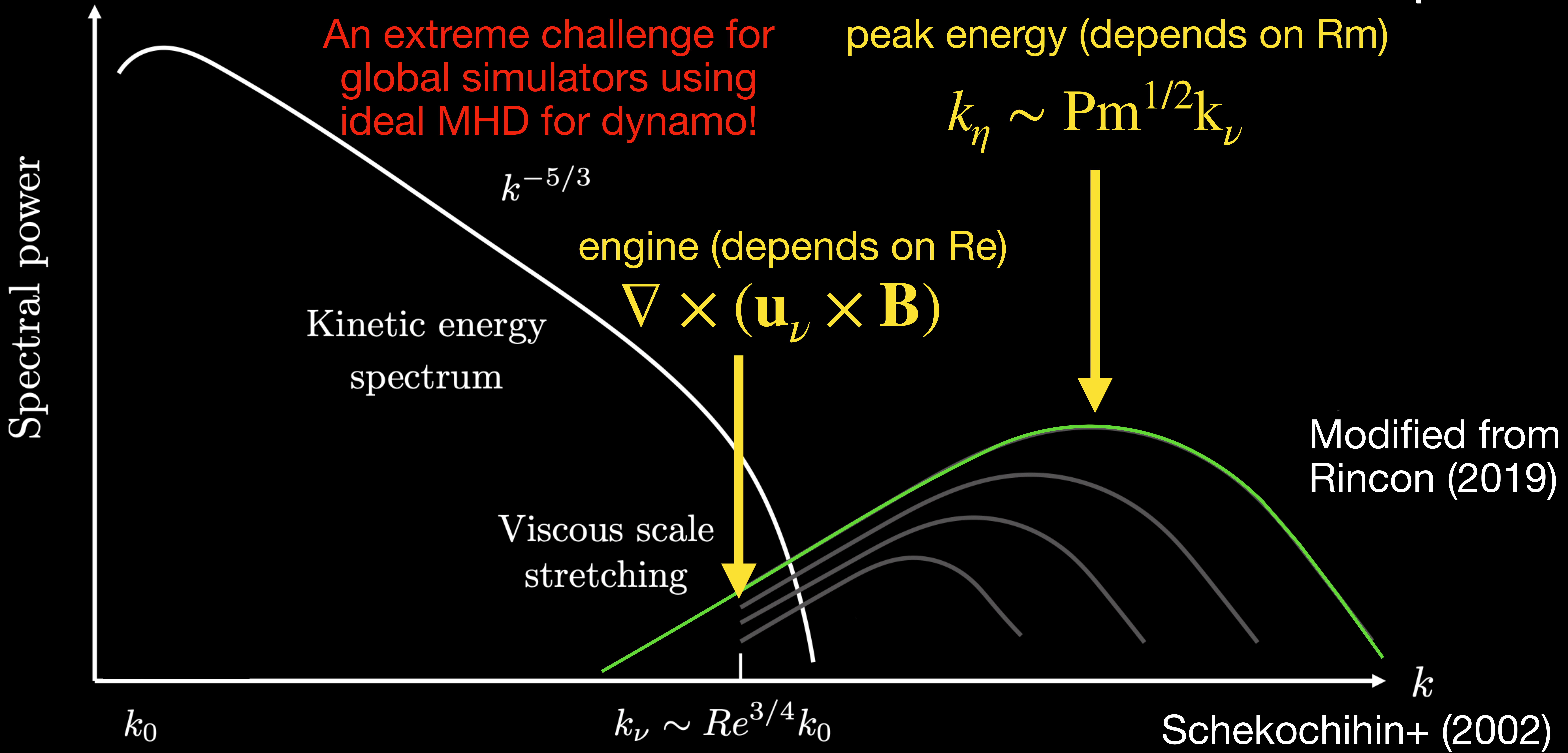


Kriel, **Beattie**+ (2025). *Fundamental scales I: the kinematic stage of the subsonic dynamo*

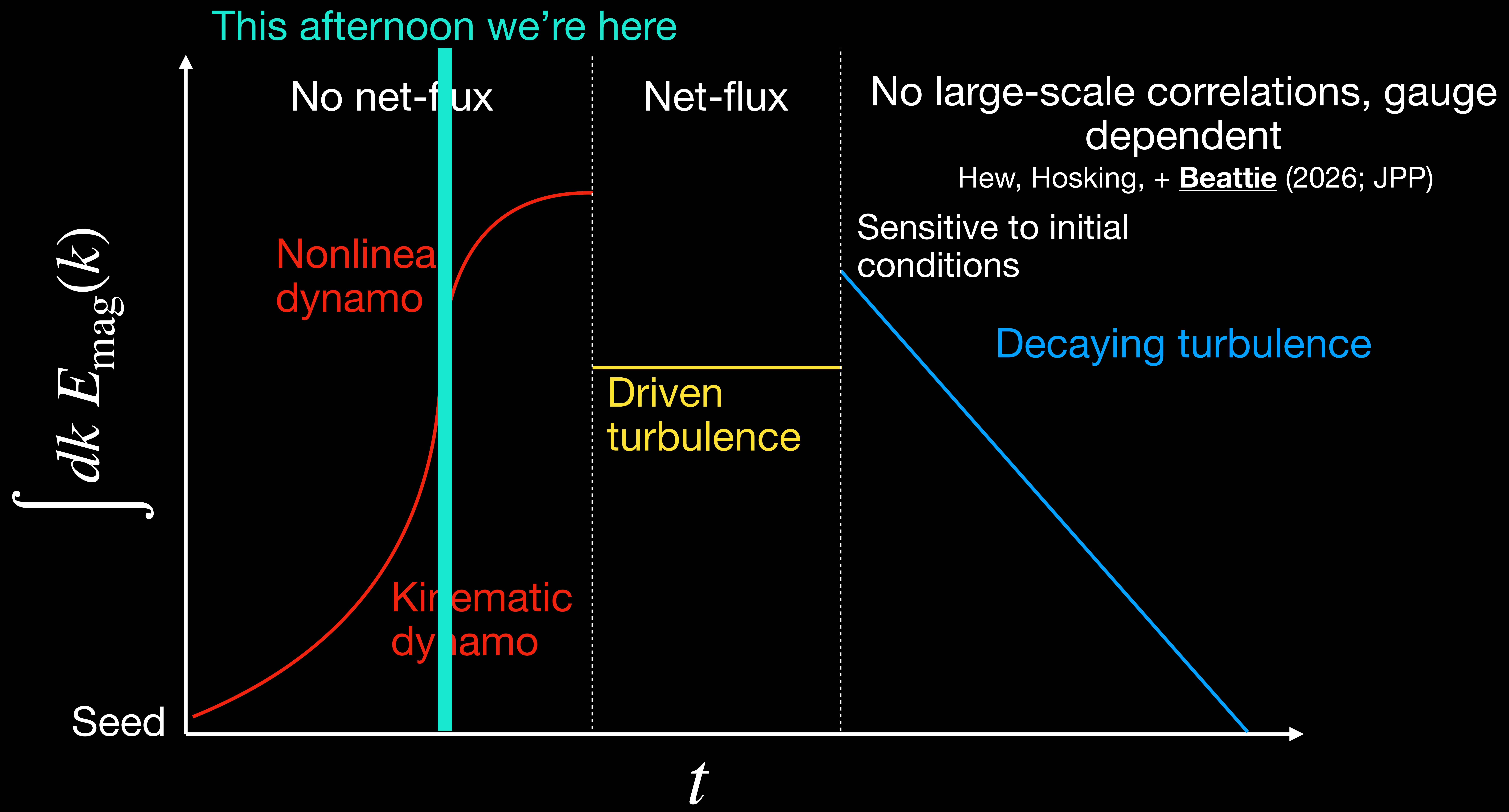
Turbulence dynamo

Viscosity and resistivity matter!

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

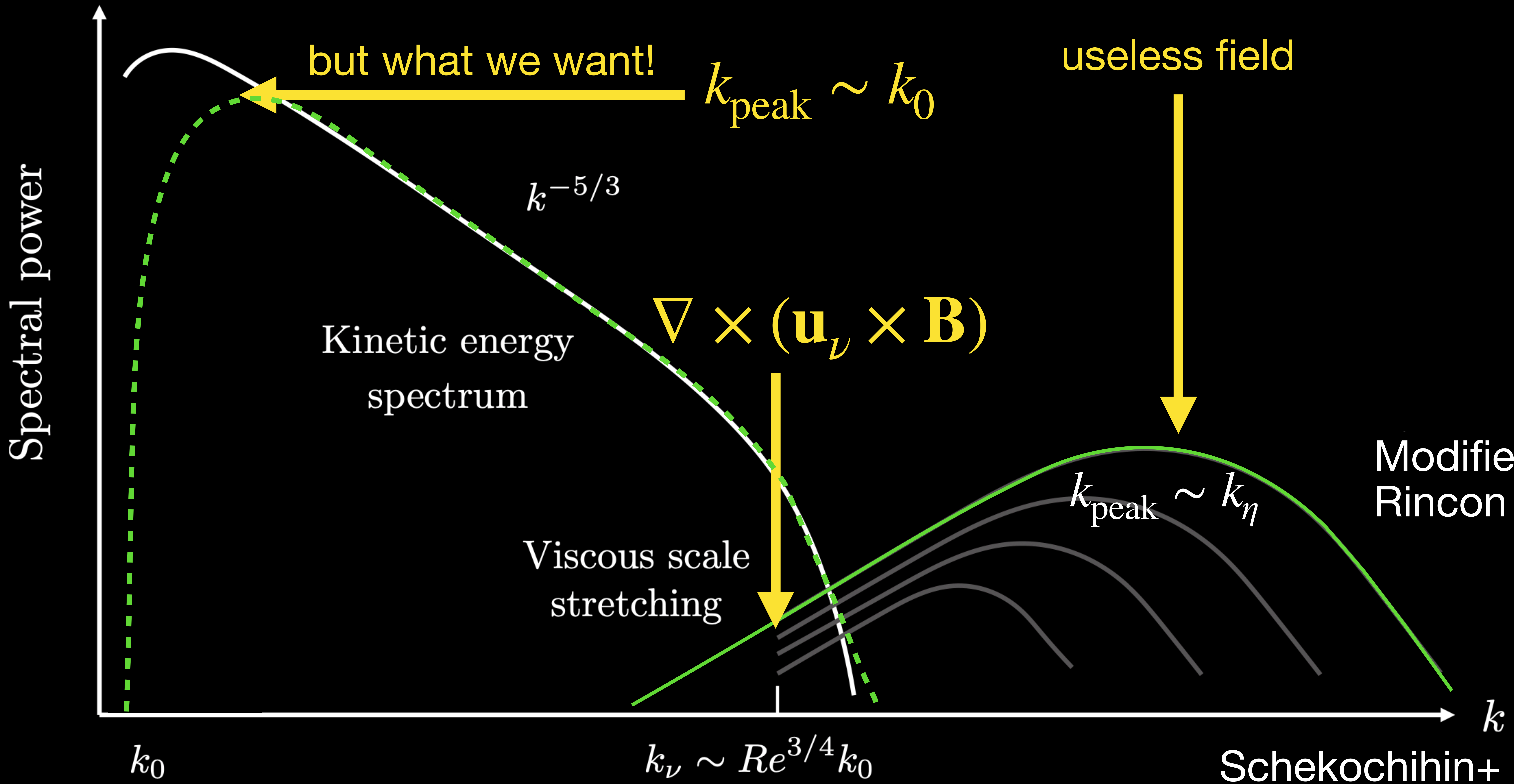


The lifecycle of a high-k mode spans across a patchwork quilt of theories



Turbulence dynamo

If the turbulence dynamo only generates small-scale fields it's useless

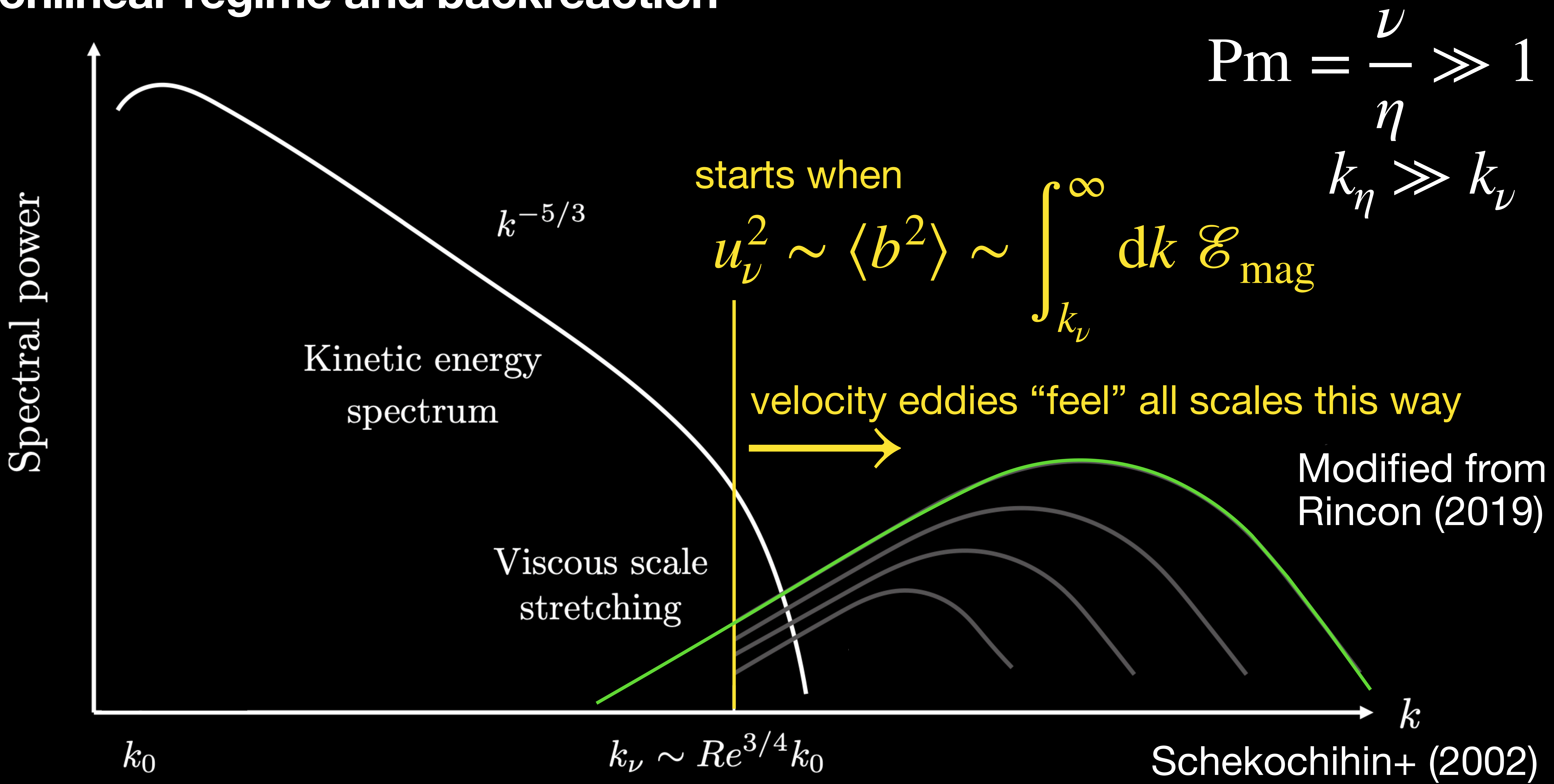


Modified from Rincon (2019)

Schekochihin+ (2002)

Turbulence dynamo

Nonlinear regime and backreaction



Turbulence dynamo

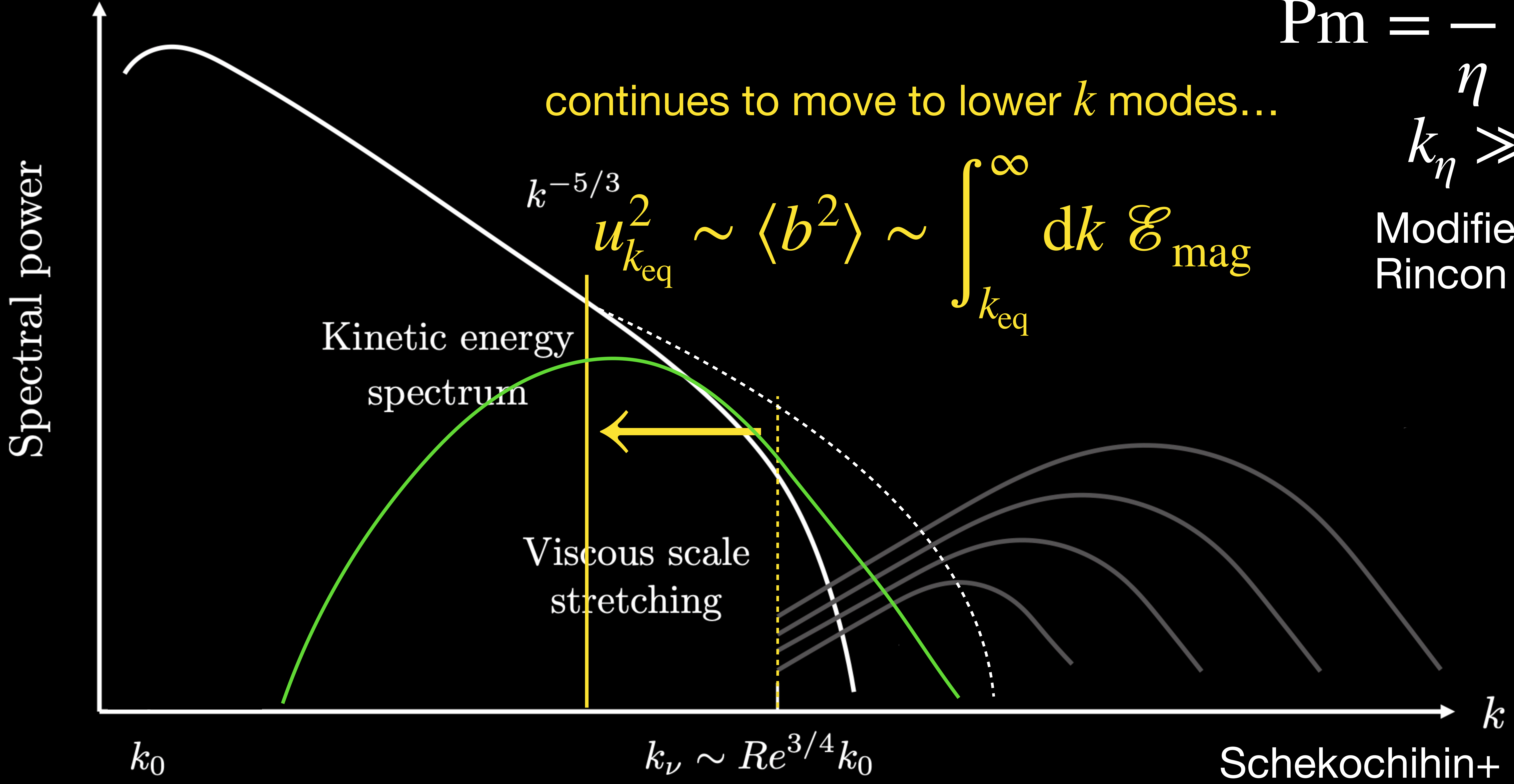
Nonlinear regime and backreaction

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

Modified from Rincon (2019)

continues to move to lower k modes...

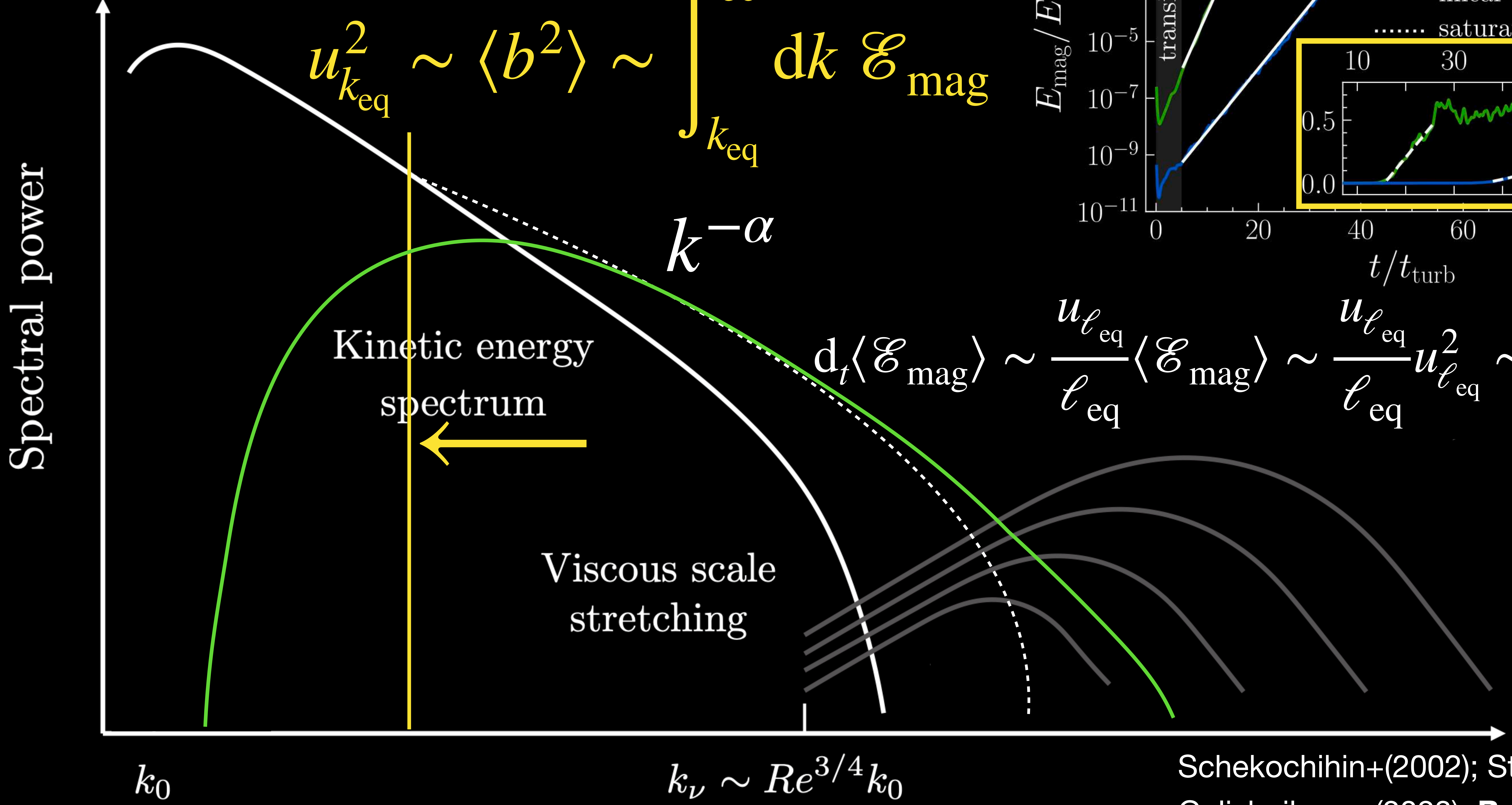


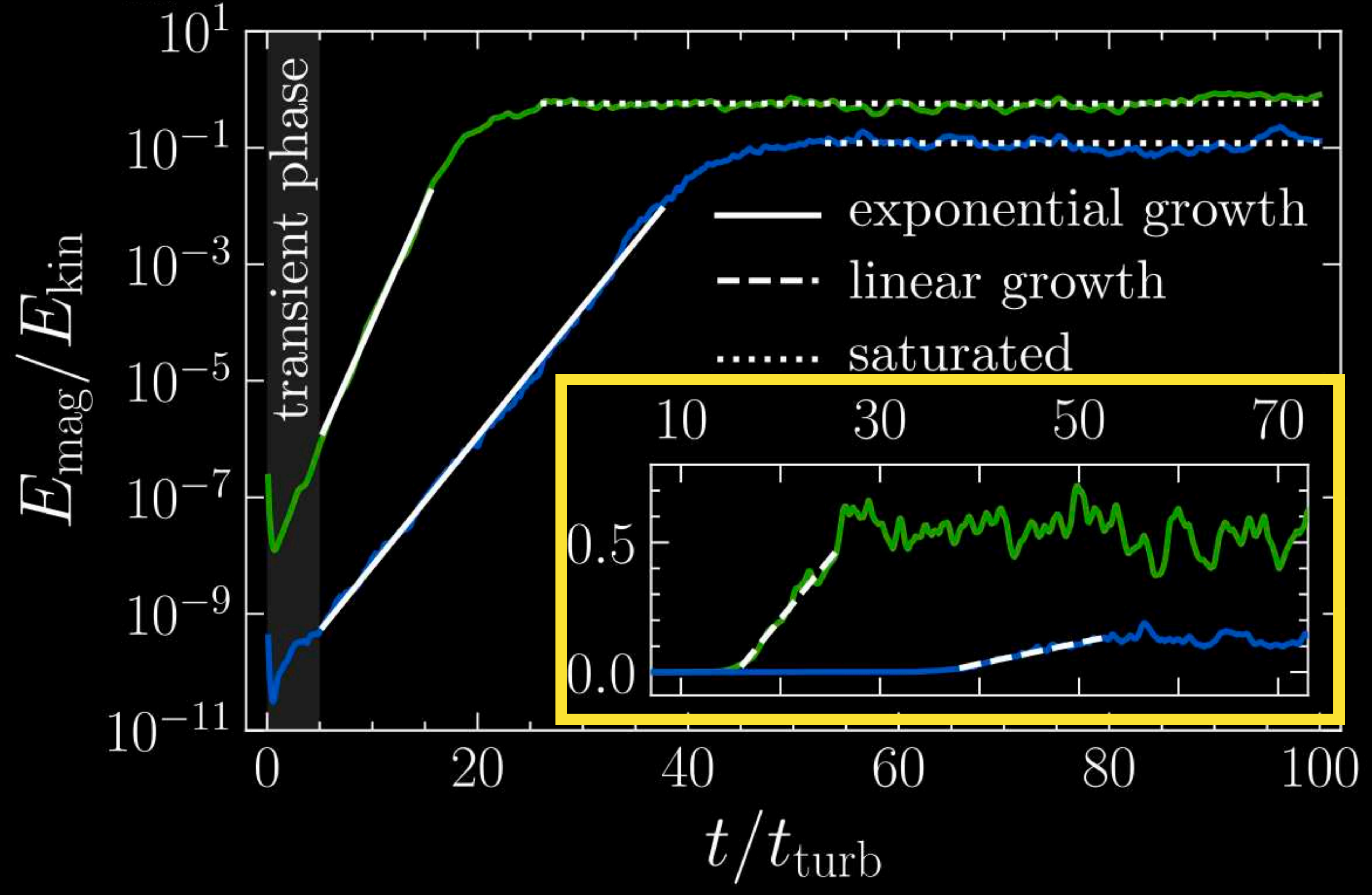
Schekochihin+ (2002)

Turbulence dynamo

Nonlinear regime and backreaction

$$u_{k_{\rm eq}}^2 \sim \langle b^2 \rangle \sim \int_{k_{\rm eq}}^{\infty} {\rm d}k\, \mathcal{E}_{\rm mag}$$





$$d_t \langle \mathcal{E}_{\rm mag} \rangle \sim \frac{u_{\ell_{\rm eq}}}{\ell_{\rm eq}} \langle \mathcal{E}_{\rm mag} \rangle \sim \frac{u_{\ell_{\rm eq}}}{\ell_{\rm eq}} u_{\ell_{\rm eq}}^2 \sim \varepsilon_{\rm eq} \sim \varepsilon,$$

Schekochihin+(2002); St-Onge+(2020)
Galishnikova+(2023); **Beattie**+(2025)

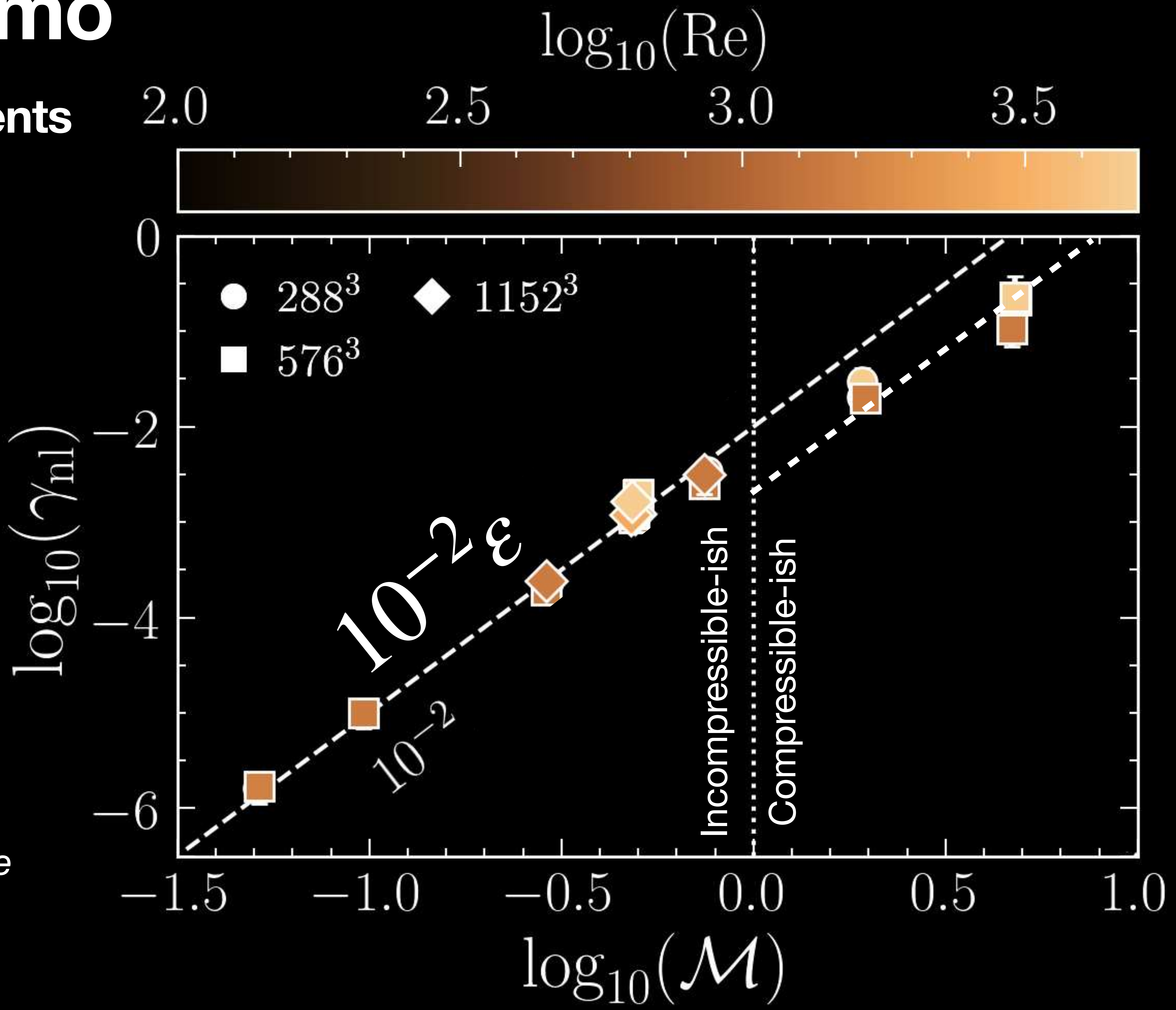
Turbulence dynamo

Nonlinear regime: fed by flux, not velocity gradients

The engine of the nonlinear dynamo is the energy flux of the cascade

$$\langle \mathscr{E}_{\mathrm{mag}} \rangle \sim \varepsilon t$$

Kriel, **Beattie**+ (2026). *The universal growth of magnetic energy during the nonlinear phase of subsonic and supersonic small-scale dynamos*



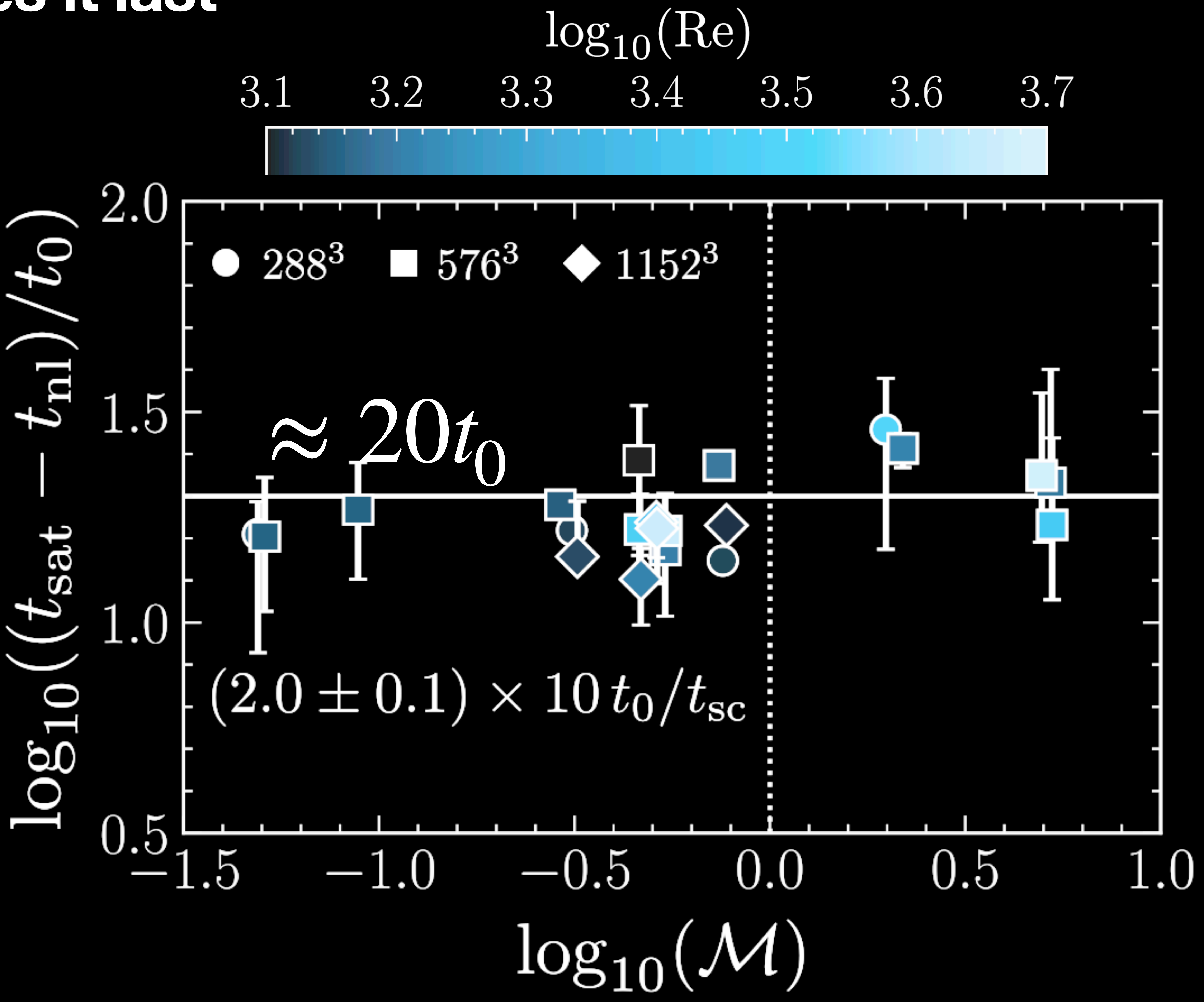
Turbulence dynamo

Nonlinear regime: how long does it last

$$\Delta t_{\rm nl} \approx 20 t_0$$

universally

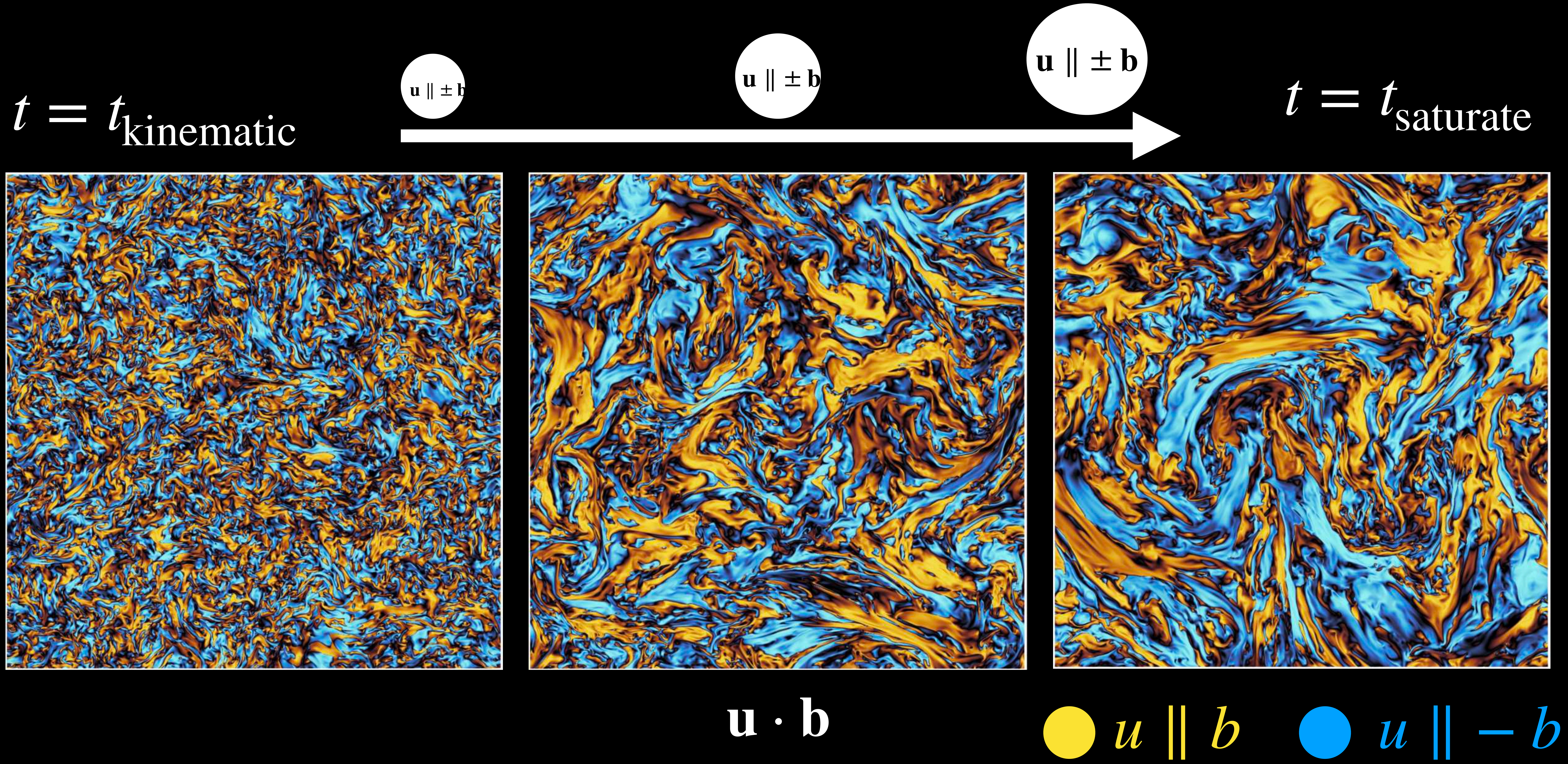
Somewhat mysterious to us at the moment. Implies efficiency is universal for super or subsonic nonlinear regime.



Kriel, **Beattie**+ (2026). *The universal growth of magnetic energy during the nonlinear phase of subsonic and supersonic small-scale dynamos*

Turbulence dynamo

Nonlinear regime: what does the backreaction look like?

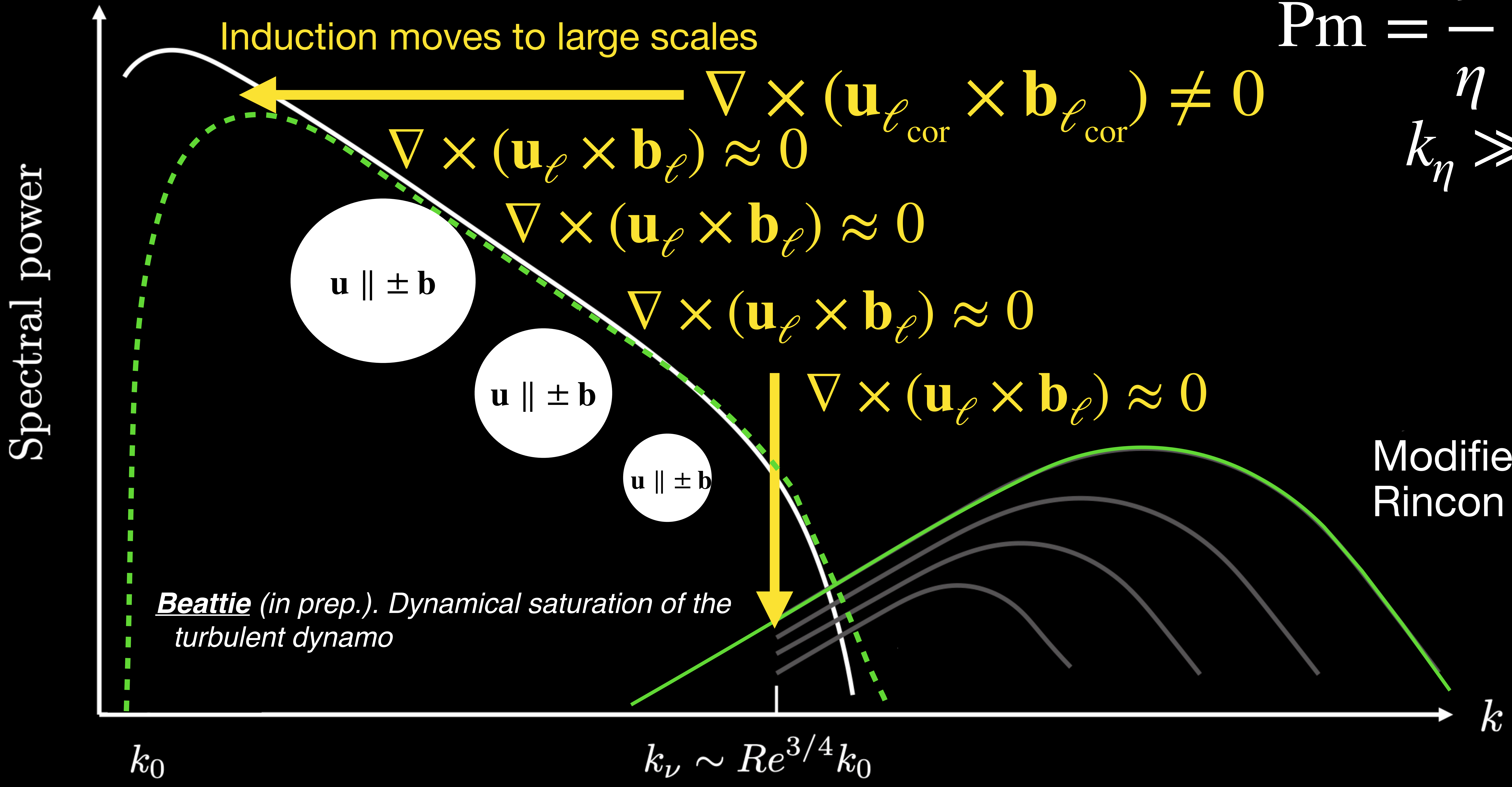


Turbulence dynamo

Saturation through alignment

$$\mathrm{Pm} = \frac{\nu}{\eta} \gg 1$$

$$k_\eta \gg k_\nu$$

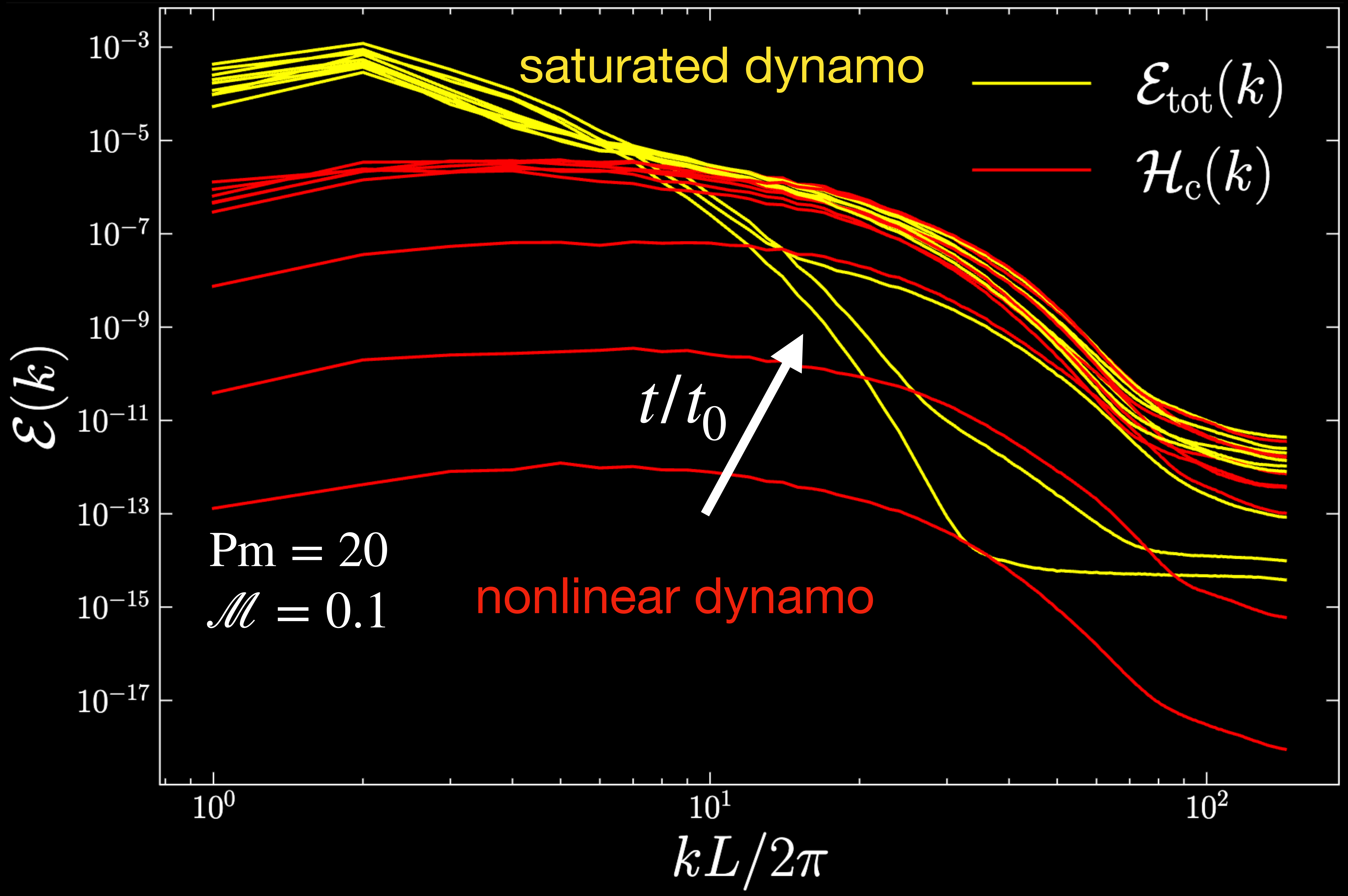


Beattie *(in prep.). Dynamical saturation of the turbulent dynamo*

Modified from Rincon (2019)

Turbulence dynamo

Saturation through alignment

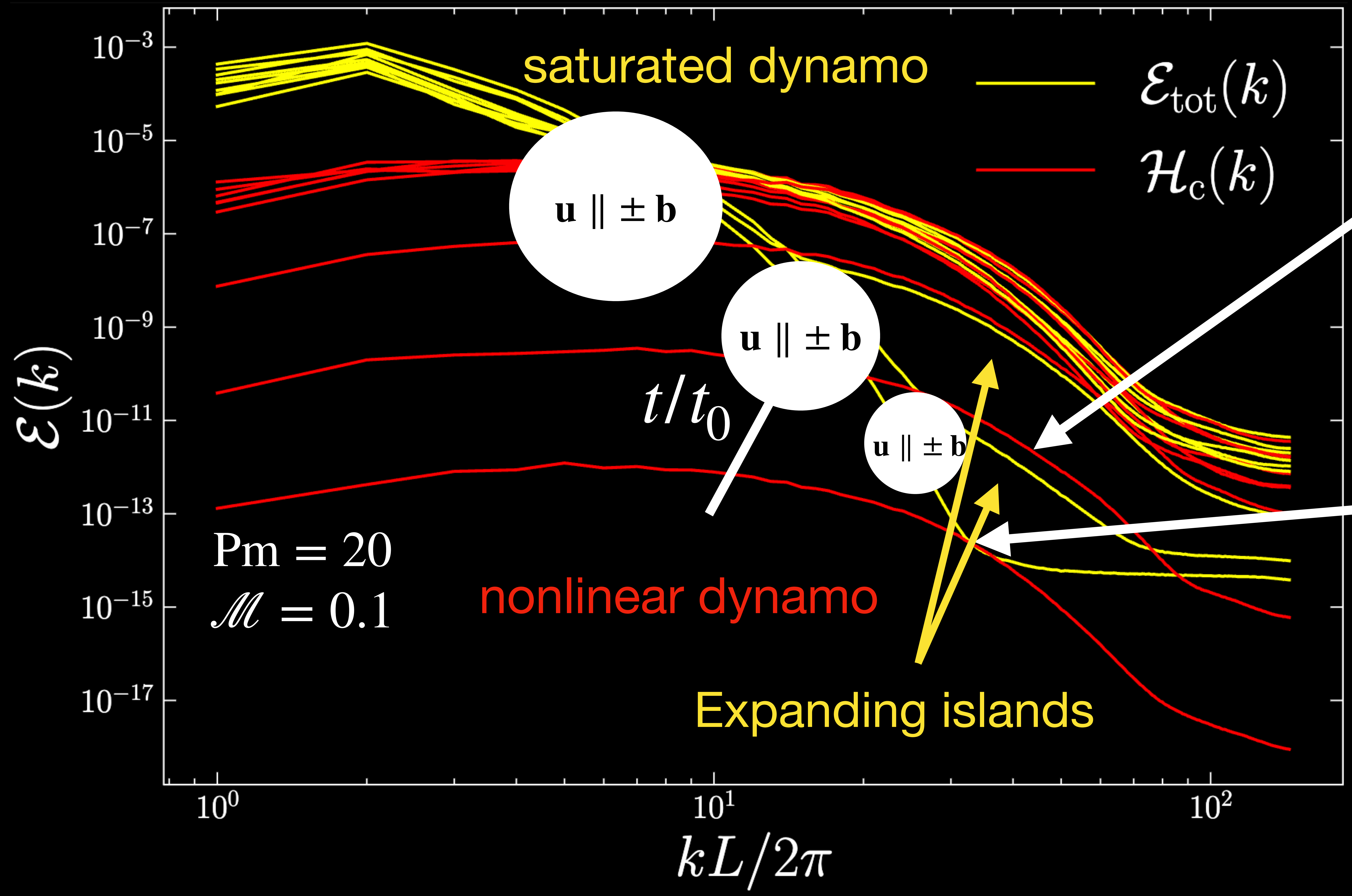


$$\mathscr{H}_c(k) = \mathbf{u}(k) \cdot \mathbf{b}^\dagger(k)$$

$\mathscr{H}_c$ grows $\Longrightarrow$ aligned scales develop where $\mathscr{H}_c > \mathscr{E}_{\rm tot} \Longrightarrow$ induction turns off

The dynamical saturation of the dynamo

Saturation through alignment

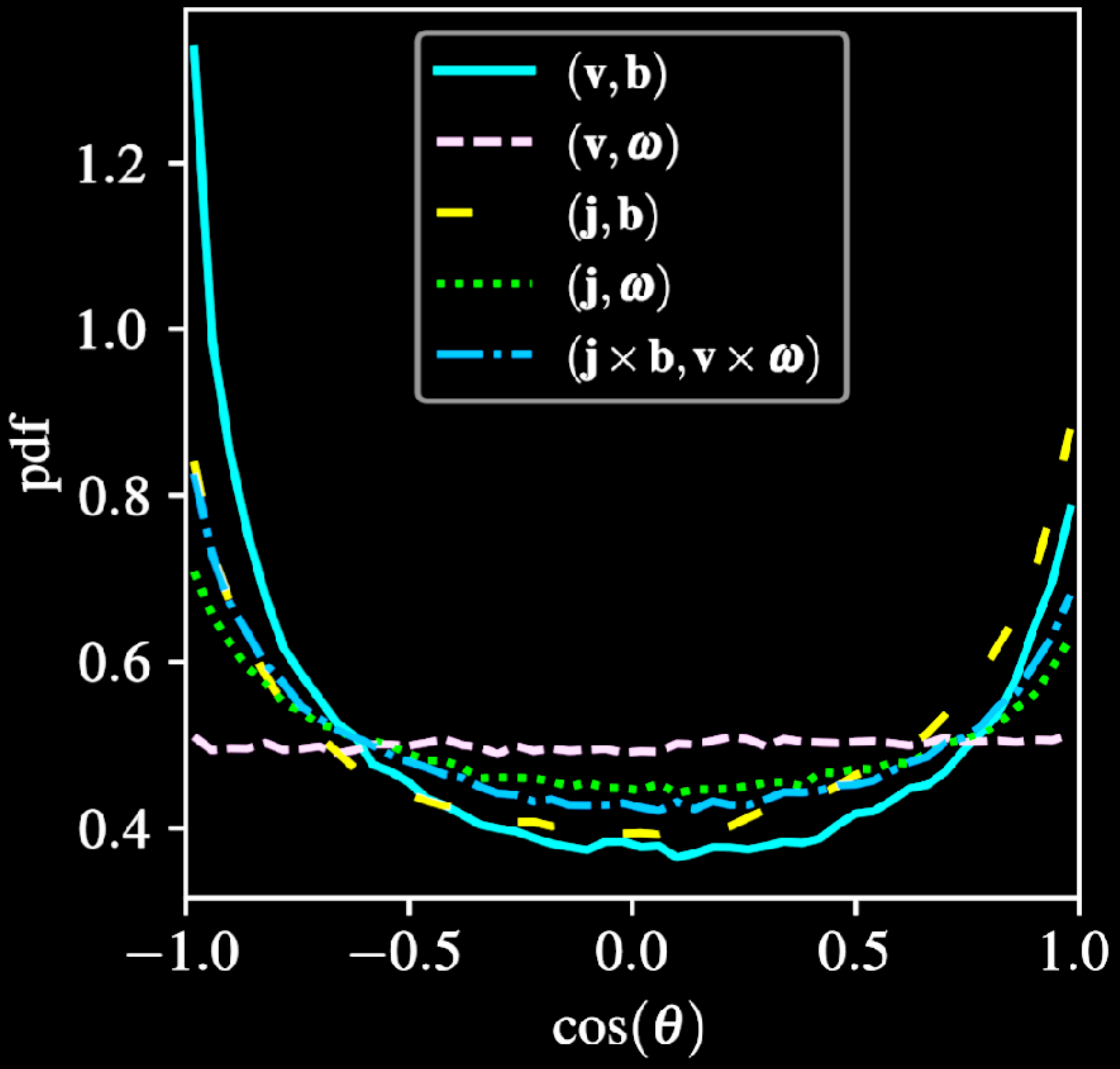


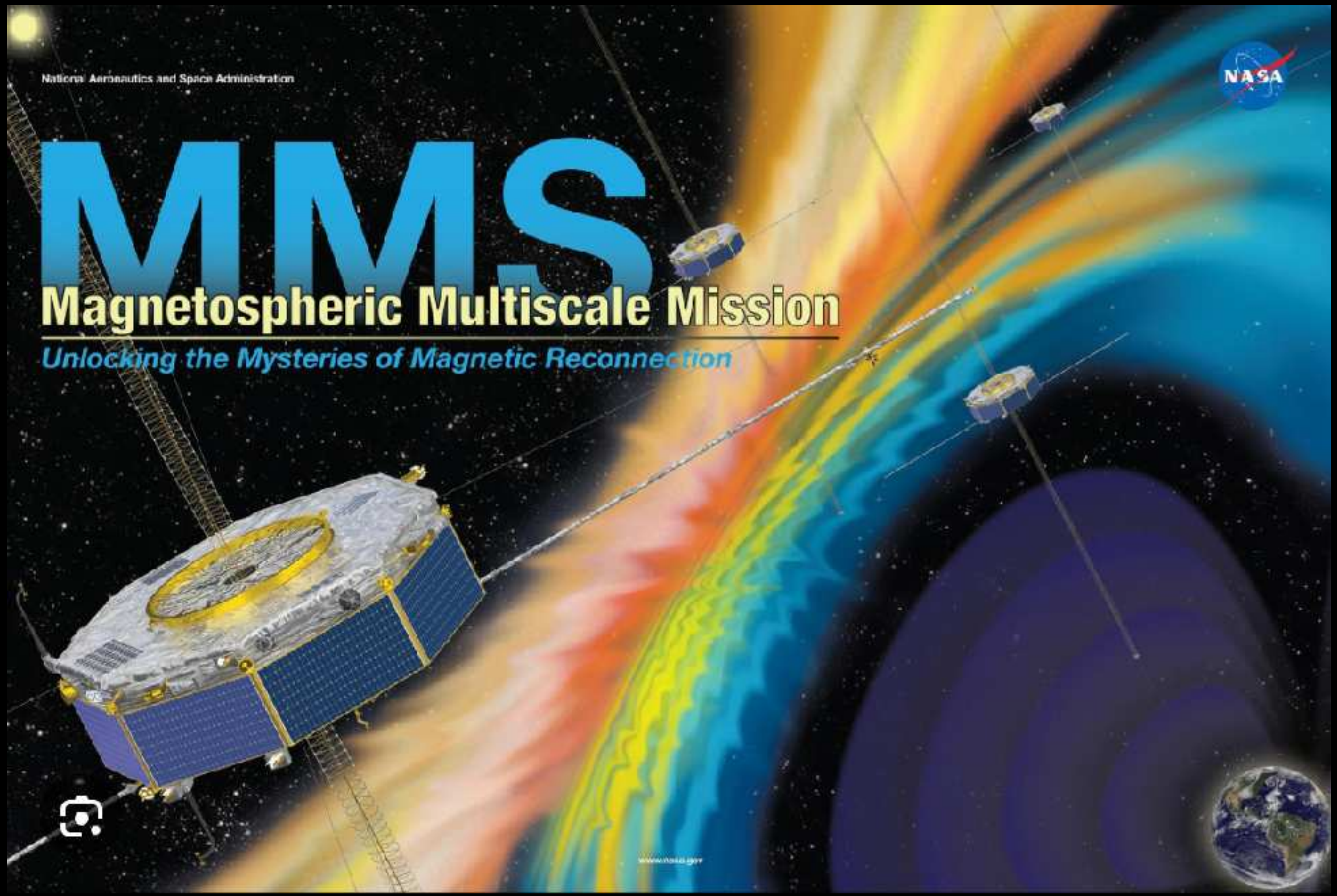
$$\mathscr{H}_c(k) = \mathbf{u}(k) \cdot \mathbf{b}^{\dagger}(k)$$

nonlinear dynamo
(backreaction turning on)

$\mathscr{H}_c$ grows $\Longrightarrow$ aligned scales develop where $\mathscr{H}_c > \mathscr{E}_{\rm tot} \Longrightarrow$ induction turns off

Patchy helicities in Earth's magnetosheath





Typical turbulent magnetosheath

$\mathscr{M}_{\mathrm{turb}} \sim 0.2$ $\beta \sim 10$

Pecora et al. (2023) · Bandyopadhyay et al. (2025) · NASA

$10{,}080^3$ magnetized supersonic turbulence simulation

Beattie, Federrath, Klessen, Cielo & Bhattacharjee

1. What is the scaling of the energy cascade in compressible MHD turbulence with no net flux?
2. How are the characteristic scales organized in the compressible supersonic turbulence?
3. What are the saturation physics of the compressible turbulent dynamo?

PI of a three total 190million core-hour projects on superMUC-NG

ILES of compressible MHD turbulence

Turbulence: $\sigma_V / c_s \approx 4,\ \ell_0 = L/2$

Magnetic fields: $B = b_{\text{turb}},\ \mathcal{M}_A \approx 2$

Three experiments for convergence tests:

- **LOW-RES:** $2{,}520^3$ (0.3Mcore-h, 8,640cores)
- **MID-RES:** $5{,}040^3$ (4.0Mcore-h, 34,560cores)
- **HIGH-RES:** $10{,}080^3$ (80.0Mcore-h, 148,240cores)

$6\,\text{PB}$ in data products $\quad \text{Rm} \sim \text{Re} \gtrsim 10^6, \quad \text{Pm} \sim 1-2$

ACCEPTED PAPER

Local relaxation and scale-dependent alignment in compressible, magnetized turbulence

James R. Beattie and Amitava Bhattacharjee

Phys. Rev. Lett. - **Accepted** 10 September, 2026

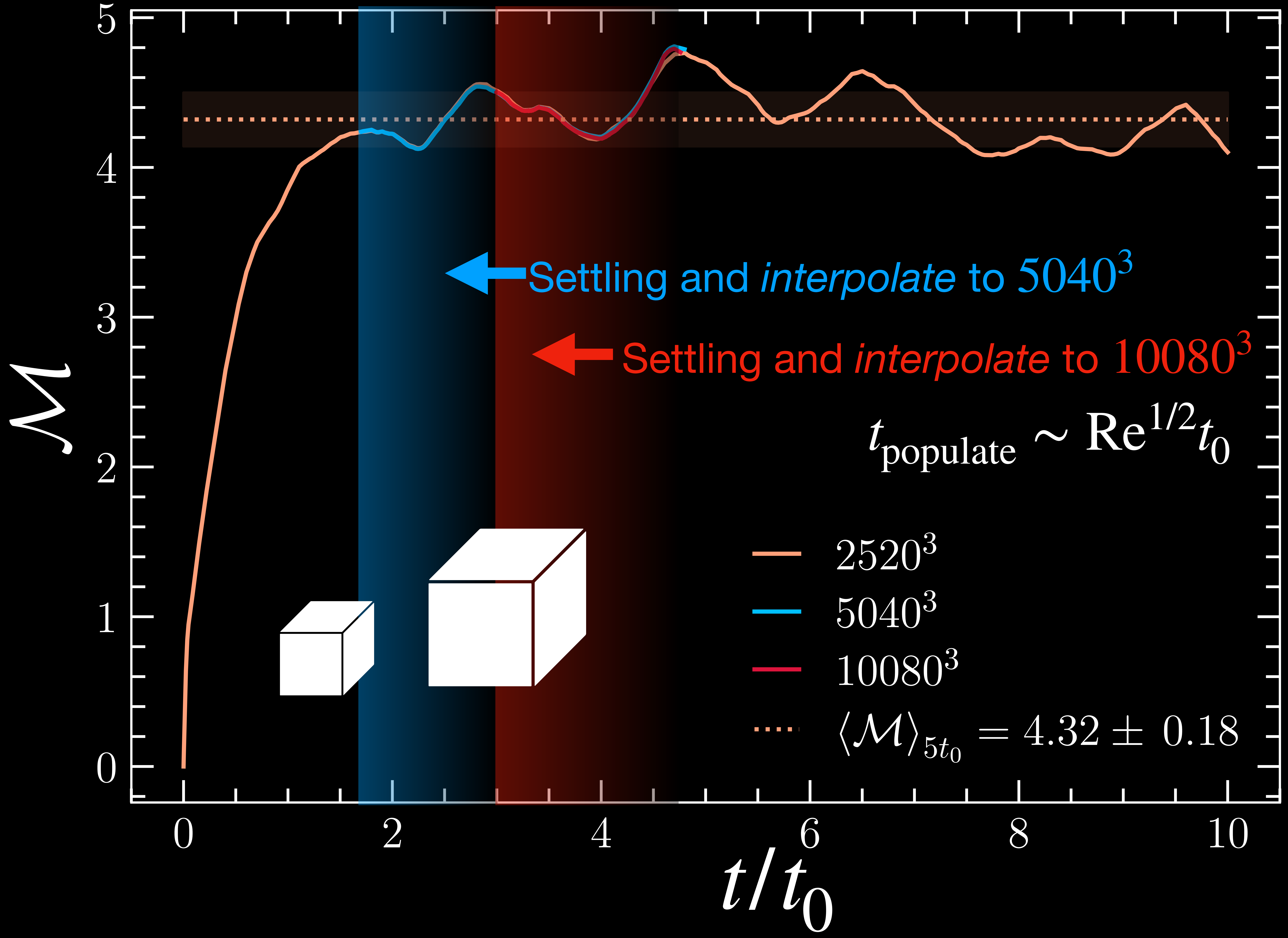
Export Citation

Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium

Beattie & Bhattacharjee, 2026 (PRL). Local relaxation and scale-dependent alignment in compressible, magnetized turbulence

Volume integral Quantities

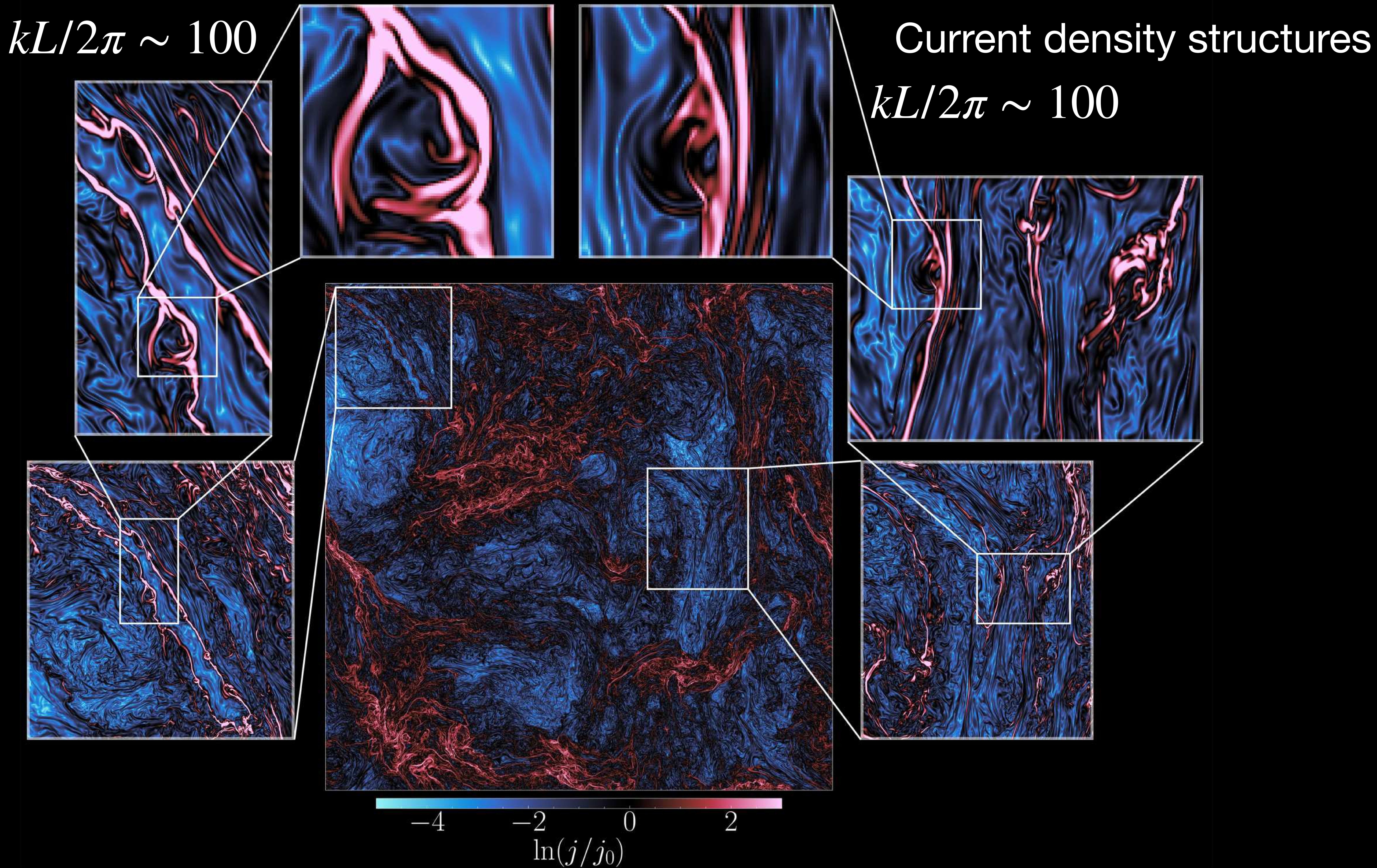
$$\mathscr{M} = \left\langle \frac{u^2}{c_s^2} \right\rangle_{\mathscr{V}}^{1/2}$$



Interpolation to generate ICs for successively higher resolution experiments

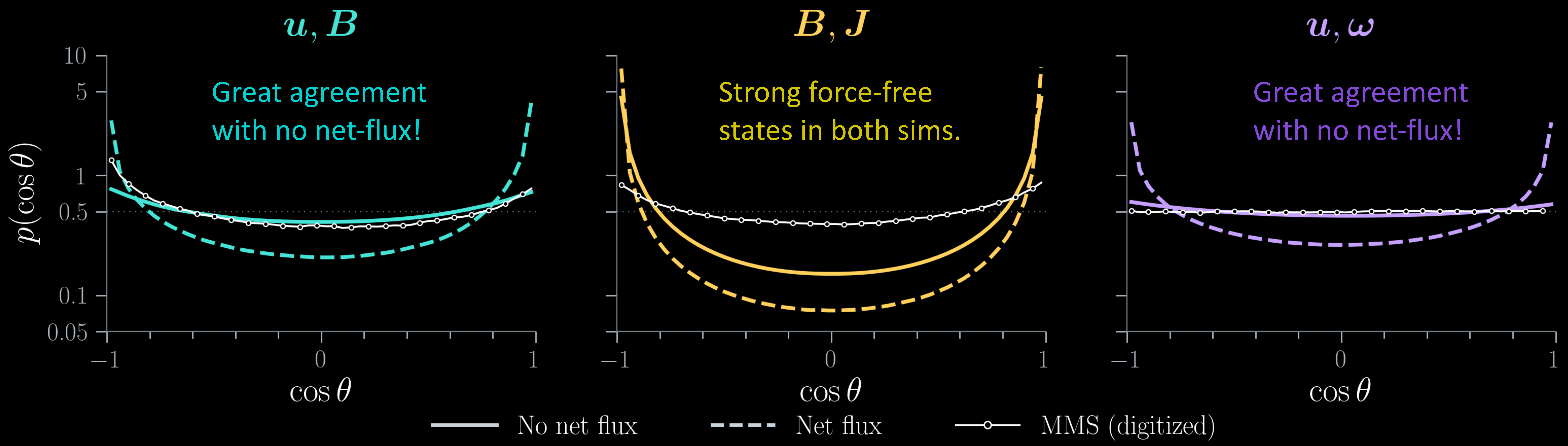
Beattie+ 2025 (*Nature Astronomy*). The spectrum of magnetized turbulence in the interstellar medium


$\ln(\rho/\rho_0)$


$kL/2\pi \sim 100$
Current density structures
$kL/2\pi \sim 100$
-4
-2
0
2
$\ln(j/j_0)$

Patchy helicities in simulation & magnetosheath

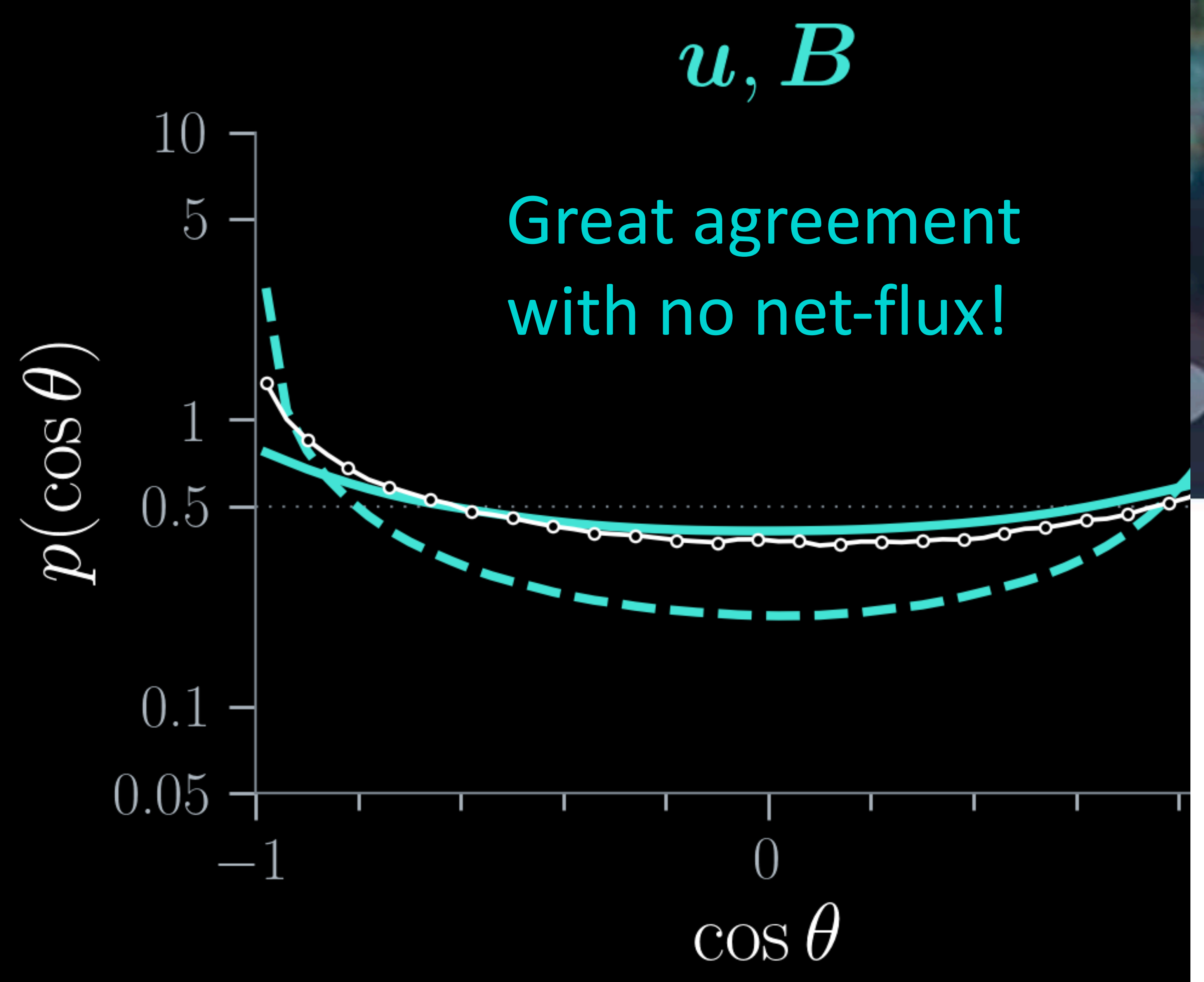
Pecora et al. (2023) for MMS data.

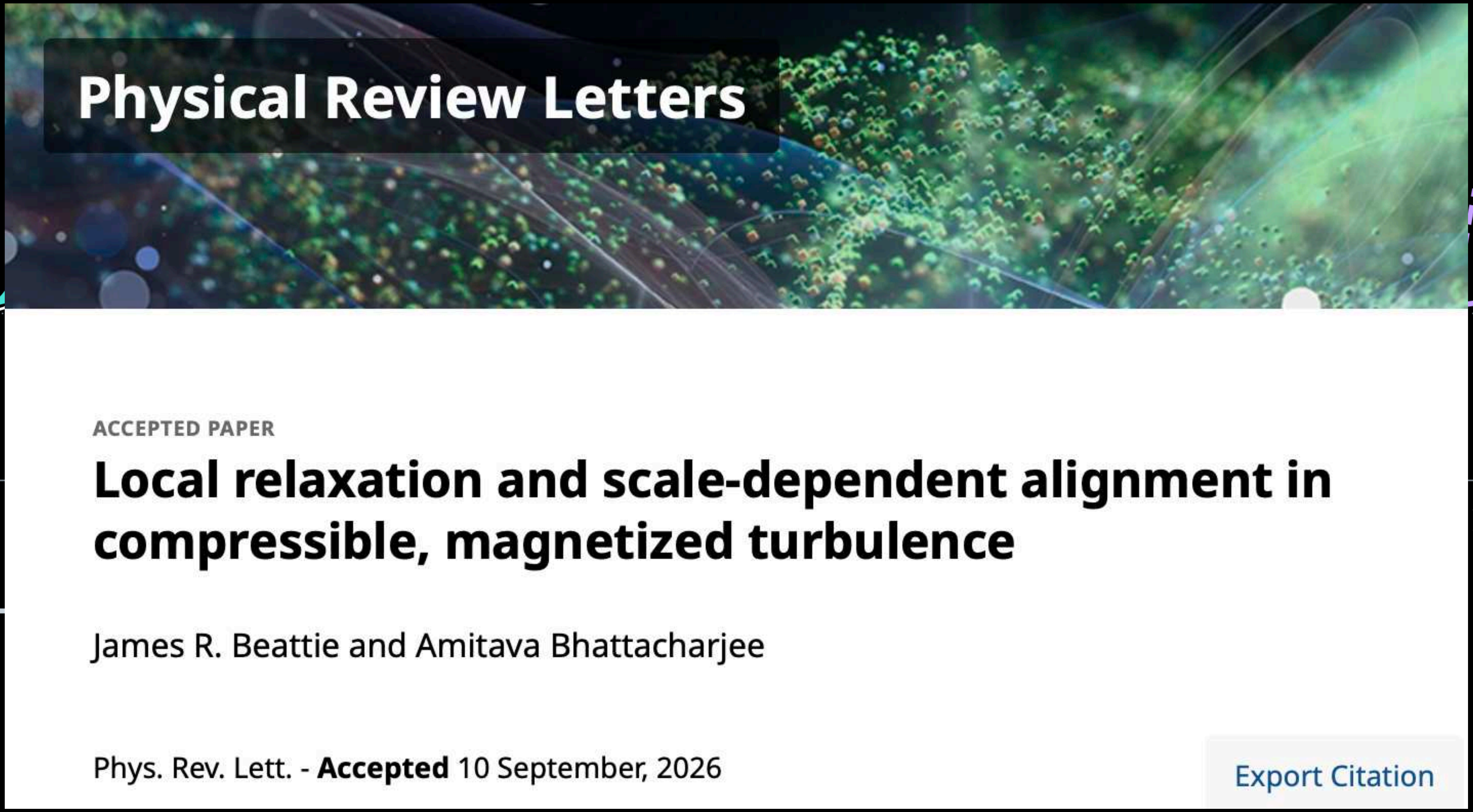


Patchy helicities in simulation & magnetosheath

Pecora et al. (2023) for MMS data.

Why?… have to invite me back :)




Physical Review Letters

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Export Citation

Conclusions for this afternoon talk

1. Kinematic and nonlinear theory at the level of integral energy work really well, with subtle but expected changes between the subsonic and supersonic regimes.

2. The alignment ends up playing an extremely important and generic role in saturating the turbulent dynamo…

    1. Potentially extremely universal, setting the saturation across alll of the plasmas I introduced at the start of the talk!

    2. Come to Muni’s plasma class to hear more about the simulation and saturation.